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CHAPTER 17

Statistics for Quality: Control and Capability

Introduction

Quality is a broad concept. Often, it refers to a degree or grade of excellence. For example, you may feel that a restaurant serving filet mignon is a higher-quality establishment than a fast-food outlet that primarily serves hamburgers. You may also consider a name-brand sweater of higher quality than one sold at a discount store.

In this chapter, we consider a narrower concept of quality: *consistently meeting standards appropriate for a specific product or service*. The fast-food outlet, for example, may serve high-quality hamburgers. The hamburgers are freshly grilled and served promptly at the right temperature every time you visit. Similarly, the discount store sweaters may be high quality because they are consistently free of defects and the tight knit helps them keep their shape wash after wash. Also, the restaurant filet mignon may be poor quality because a medium rare order is often served well done.

Statistically minded management can assess this concept of quality through sampling. For example, the fast-food outlet could sample hamburgers and measure the time from order to being served as well as the temperature and tenderness of the burgers. This chapter discusses the methods used to monitor the quality of a product or service and effectively detect changes in the process that may affect its quality.

Use of Data to Assess Quality

Organizations are (or ought to be) concerned about the quality of the products and services they offer. What they don't know about quality can hurt them: rather than make complaints that an alert organization could use as warnings, customers often simply leave when they feel they are receiving poor quality. A key to maintaining and improving quality is systematic use of *data* in place of intuition or anecdotes. Here are two data-driven decision making examples.

EXAMPLE 17.1

Membership renewal process. Sometimes data that are routinely produced make a quality problem obvious. The internal financial statements of a professional society showed that hiring temporary employees to enter membership data was causing expenditures above budgeted levels each year during the several months when memberships were renewed. Investigation led to two actions. Membership renewal dates were staggered across the year to spread the workload more evenly. More important, outdated and inflexible data entry software was replaced by a modern system that was much easier to use. Result: permanent employees could now process renewals quickly, eliminating the need for temps and also reducing member complaints.

EXAMPLE 17.2

Response time process. Systematic collection of data helps an organization to move beyond dealing with obvious problems. Motorola measures the performance of its services and manufactured products. They track, for example, the average time from a customer's call until the problem is fixed, month by month. The trend should be steadily downward as ways are found to speed response.

LOOK BACK
time plot, p. 22
regression line, p. 107
comparative experiments, p. 173
sampling distributions, p. 286

Because using data is a key to improving quality, statistical methods have much to contribute. Simple tools are often the most effective. Motorola's service centers calculate mean response times each month and make a time plot. A scatterplot—and perhaps a regression line—can show how the time to answer telephone calls to a corporate call center influences the percent of callers who hang up before their calls are answered. The design of a new product such as a smartphone may involve interviewing samples of consumers to learn what features they want included and using randomized comparative experiments to determine the best interface.

This chapter focuses on just one aspect of statistics for improving quality: *statistical process control.* The techniques are simple and are based on sampling distributions, but the underlying ideas are important and a bit subtle.

17.1 Processes and Statistical Process Control

When you complete this section, you will be able to:

- Describe a process using a flowchart and a cause-and-effect diagram.
- Explain what is meant by a process being in control by distinguishing common and special cause variation.
- Compute the center line and control limits for an $\bar{x}$ chart.
- Compute the center line and control limits for an s chart.
- Contrast the $\bar{x}$ and s charts in terms of what they monitor and which should be interpreted first.
- Use the $\bar{x}$ and s chart for process monitoring.

In thinking about statistical inference, we distinguish between the *sample* data we have in hand and the wider *population* that the data represent. We hope to use the sample to draw conclusions about the population. In thinking about quality improvement, it is often more natural to speak of *processes* rather than populations. This is because work is organized in processes. Here are some examples:

- Processing an application for admission to a university and deciding whether or not to admit the student.
- Reviewing an employee's expense report for a business trip and issuing a reimbursement check.
- Hot forging to shape a billet of titanium into a blank that, after machining, will become part of a medical implant for hip, knee, or shoulder replacement.

Each of these processes is made up of several successive operations that eventually produce the output—an admission decision, a reimbursement check, or a metal component.

PROCESS

A **process** is a chain of activities that turns inputs into outputs.

We can accommodate processes in our sample-versus-population framework: think of the population as containing all the outputs that would be produced by the process if it ran forever in its present state. The outputs produced today or this week are a sample from this population. Because the population doesn't actually exist now, it is simpler to speak of a process and of recent output as a sample from the process in its present state.

Describing processes

The first step in improving a process is to understand it. If the process is at all complex, even the people involved with it may not have a full picture of how the activities interact in ways that influence quality. A brainstorming session is in order: bring people together to gain an understanding of the process.

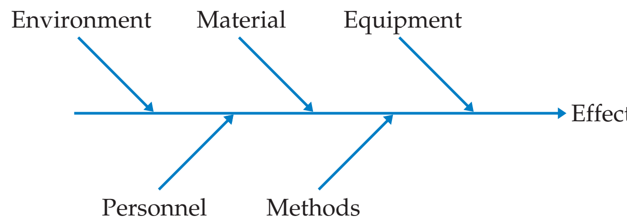

FIGURE 17.1 An outline for a cause-and-effect diagram. Group causes under these main headings in the form of branches.

This understanding is often presented graphically using two simple tools: flowcharts and cause-and-effect diagrams. A **flowchart** is a picture of the stages of a process. Many organizations have formal standards for making flowcharts. We will informally illustrate their use in an example and not insist on a specific format. A **cause-and-effect diagram** organizes the logical relationships between the inputs and stages of a process and an output. Sometimes, the output is successful completion of the process task; sometimes it is a quality problem that we hope to solve. A good starting outline for a cause-and-effect diagram appears in Figure 17.1. The main branches organize the causes and serve as a skeleton for detailed entries. You can see why these are sometimes called "fishbone diagrams." Once again, we will illustrate the diagram by example rather than insist on a specific format.[1]

flowchart

cause-and-effect diagram

EXAMPLE 17.3

Michael Rosenfeld/Getty Images

Flowchart and cause-and-effect diagram of a hot-forging process. Hot forging involves heating metal to a plastic state and then shaping it by applying thousands of pounds of pressure to force the metal into a die (a kind of mold). Figure 17.2 is a flowchart of a typical hot-forging process.[2]

A process improvement team, after making and discussing this flowchart, came to several conclusions:

- Inspecting the billets of metal received from the supplier adds no value. Insist that the supplier be responsible for the quality of the material. This then eliminates the inspection step.
- If possible, buy the metal billets already cut to rough length and deburred by the supplier. This would eliminate the cost of preparing the raw material.
- Heating the metal billet and forging (pressing the hot metal into the die) are the heart of the process. The company should concentrate attention here.

The team then prepared a cause-and-effect diagram (Figure 17.3, page 17-6) for the heating and forging part of the process. The team members shared their specialist knowledge of the causes in their area, resulting in a more complete picture than any one person could produce. Figure 17.3 is a simplified version of the actual diagram. We have given some added detail for the "hammer stroke" branch under "equipment" to illustrate the next level of branches. Even this requires some knowledge of hot forging to understand. Based on detailed discussion of the diagram, the team decided what variables to measure and at what stages of the process to measure them. Producing well-chosen data is the key to improving the process.

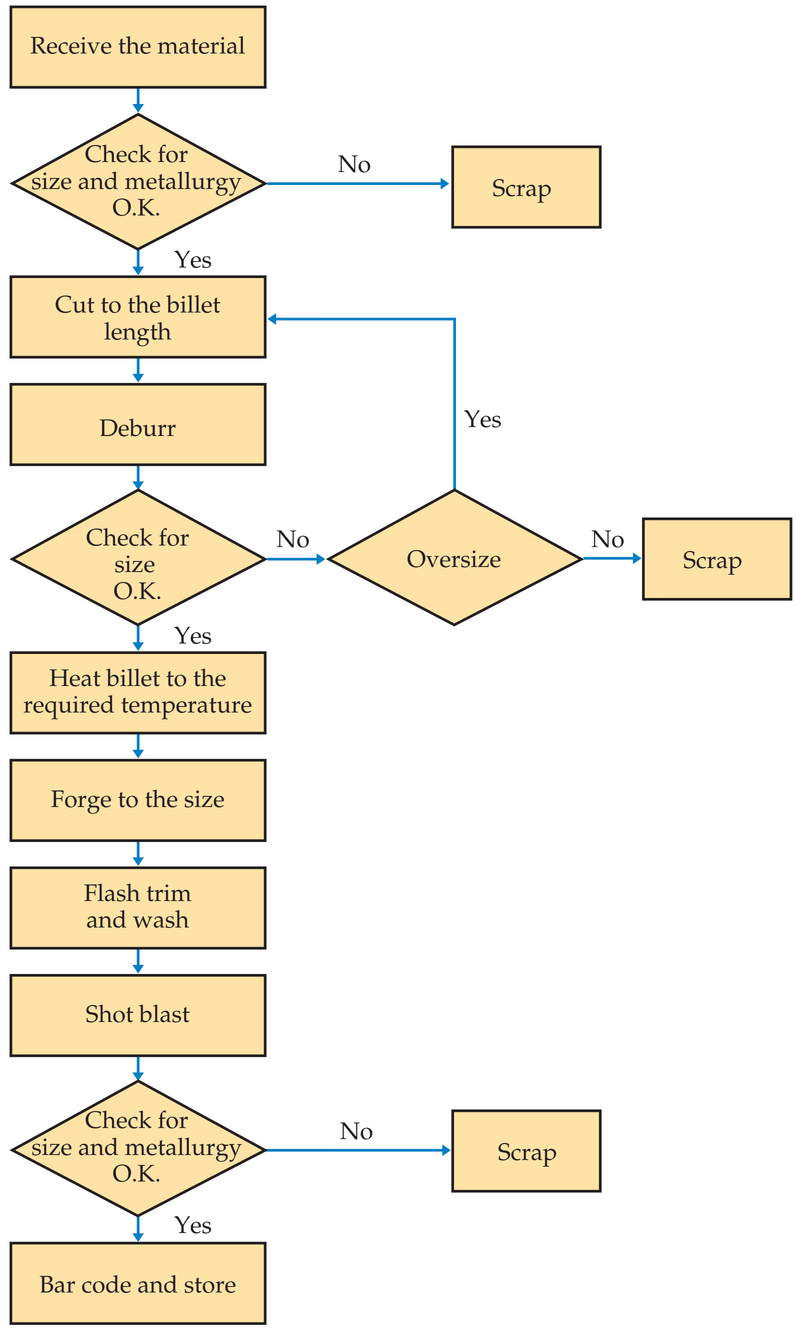

FIGURE 17.2 Flowchart of the hot-forging process, Example 17.3. Use this as a model for flowcharts: decision points appear as diamonds, and other steps in the process appear as rectangles. Arrows represent flow from step to step.

We will apply statistical methods to a series of measurements made on a process. Deciding what specific variables to measure is an important step in quality improvement. Often, we use a "performance measure" that describes an output of a process. A company's financial office might record the percent of errors that outside auditors find in expense account reports or the

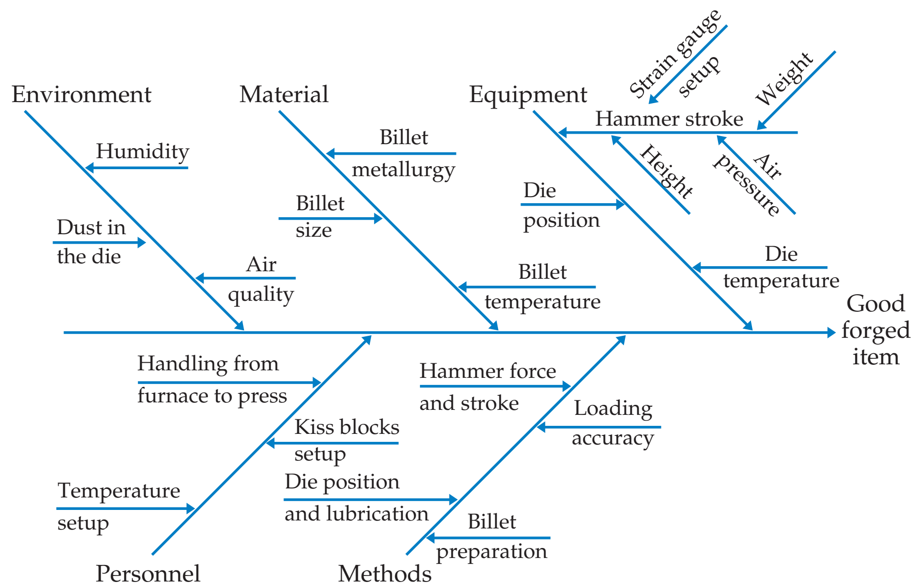

FIGURE 17.3 Simplified cause-and-effect diagram of the hot-forging process, Example 17.3. Good cause-and-effect diagrams require detailed knowledge of the specific process.

number of data entry errors per week. The personnel department may measure the time to process employee insurance claims or the percent of job offers that are accepted. *In the case of complex processes, it is wise to measure key steps within the process rather than just final outputs.* The process team in Example 17.3 might recommend that the temperature of the die and of the billet be measured just before forging.

USE YOUR KNOWLEDGE

17.1 Describing your process. Choose a process that you know well, preferably from a job you have held. If you lack experience with actual business processes, choose a personal process such as making macaroni and cheese or brushing your teeth. Make a flowchart of the process. Make a cause-and-effect diagram that presents the factors that lead to successful completion of the process.

17.2 What variables to measure? Based on your description of the process in Exercise 17.1, suggest specific variables that you might measure in order to

(a) Assess the overall quality of the process.

(b) Gather information on a key step within the process.

Statistical process control

The goal of statistical process control is to make a process stable over time and then keep it stable unless planned changes are made. You might want, for example, to keep your weight constant over time. A manufacturer of machine parts wants the critical dimensions to be the same for all parts. “Constant over time” and “the same for all” are not realistic requirements. They ignore the fact that *all processes have variation.* Your weight fluctuates from day to day; the critical dimension of a machined part varies a bit from item to item; the time to process a college admission application is not the same for all applications. Variation occurs in even the most precisely made product due to small changes in the raw material, the behavior of the machine or operator, and even the temperature in the plant.

Because variation is always present, we can't expect to hold a variable exactly constant over time. The statistical description of stability over time requires that the *pattern of variation* remain stable, not that there be no variation in the variable measured.

common cause

special cause

In the language of statistical quality control, a process that is in control has only **common cause** variation. Common cause variation is the inherent variability of the process, due to many small causes that are always present. When the normal functioning of the process is disturbed by some unpredictable event, **special cause** variation is added to the common cause variation. We hope to be able to discover what lies behind special cause variation and eliminate that cause to restore the stable functioning of the process.

EXAMPLE 17.4

Common and special cause variation. Imagine yourself doing the same task repeatedly—say, folding a mailer, stuffing it into a stamped envelope, and sealing the envelope. The time to complete this task will vary a bit, and it is hard to point to any one reason for the variation. Your completion time shows only common cause variation.

Now you receive a text. You begin a text conversation, and though you continue folding and stuffing while texting, your completion time rises beyond the level expected from common causes alone. Texting adds special cause variation to the common cause variation that is always present. The process has been disturbed and is no longer in its normal and stable state.

Control charts work by distinguishing the always-present common cause variation in a process from the additional variation that suggests that the process has been disturbed by a special cause. A control chart sounds an alarm when it sees too much variation. This is accomplished through a combination of graphical and numerical descriptions of data with use of sampling distributions.

LOOK BACK

sampling distributions, p. 286

Control charts were invented in the 1920s by Walter Shewhart at the Bell Telephone Laboratories.[3] The most common application of control charts is to monitor the performance of industrial and business processes. The same methods, however, can be used to check the stability of quantities as varied as the ratings of a television show, the level of ozone in the atmosphere, and the gas mileage of your car.

STATISTICAL CONTROL

A variable that continues to be described by the same distribution when observed over time is said to be in statistical control, or simply **in control**.

Control charts are statistical tools that monitor a process and alert us when the process has been disturbed so that it is now **out of control**. This is a signal to find and correct the cause of the disturbance.

USE YOUR KNOWLEDGE

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17.3 Considering common and special cause variation. In Exercise 17.1 (page 17-6), you described a process that you know well. What are some sources of common cause variation in this process? What are some special causes that might, at times, drive the process out of control?

17.4 Examples of special cause variation in arrival times. Lex takes a 7:15 A.M. bus to campus each morning. Her apartment complex is near a major road and is two miles from campus. Her arrival time to campus varies a bit from day to day but is generally stable. Give several examples of special causes that might raise Lex's arrival time on a particular day.

$\bar{x}$ charts for process monitoring

When you first apply control charts to a process, the process may not be in control. Even if it is in control, you don't yet understand its behavior. You will have to collect data from the process, establish control by uncovering and removing special causes, and then set up control charts to maintain control. We call this the **chart setup** stage.

chart setup

Later, when the process has been operating in control for some time, you understand its usual behavior and have a long run of data from the process. You keep control charts to monitor the process because a special cause could erupt at any time. We will call this **process monitoring**.[4]

process monitoring

Although, in practice, chart setup precedes process monitoring, the big ideas of control charts are more easily understood in the process-monitoring setting. We will start there and then discuss the more complex process-improvement setting.

Consider a quantitative variable x that is an important measure of quality. The variable might be the diameter of a part, the number of envelopes stuffed in an hour, or the time to respond to a customer call. If this process is in control, the variable x is described by the same distribution over time. For now, we'll assume this distribution is Normal.

PROCESS-MONITORING CONDITIONS

The measured quantitative variable x has a **Normal distribution**. The process has been operating in control for a long period, so that we know the **process mean** $\boldsymbol{\mu}$ and the **process standard deviation** $\boldsymbol{\sigma}$ that describe the distribution of x as long as the process remains in control.

In practice, we must estimate the process mean and standard deviation from past data on the process. Under the process-monitoring conditions, we have numerous observations and the process has remained in control. The law of large numbers tells us that estimates from past data will be very close to the truth about the process. That is, at the process-monitoring stage, we can act as if we know the true values of μ and σ.

LOOK BACK
law of large numbers, p. 250

Note carefully that μ and σ describe the center and spread of our variable x *only as long as the process remains in control.* A special cause may at any time disturb the process and change the mean, the standard deviation, or both.

To make control charts, begin by taking small samples from the process at regular intervals. For example, we might measure four or five consecutive parts or the response times to four or five consecutive customer calls. There is an important idea here: *the observations in a sample are so close together in time that we can assume that the process is stable during this short period.* Variation within a single sample gives us a benchmark for the common cause variation in the process.

The process standard deviation σ refers to the standard deviation within the time period spanned by one sample. If the process remains in control, the same σ describes the standard deviation of observations across any time period. Control charts help us decide whether this is the case.

$\bar{x}$ chart

We start with the $\bar{x}$ **chart**, which is based on plotting the means of the successive samples. Here is the outline:

1. Take samples of size n from the process at regular intervals. Plot the sample means $\bar{x}$ against the order in which the samples were taken.

2. We know that the sampling distribution of $\bar{x}$ under the process-monitoring conditions is Normal with mean μ and standard deviation $\sigma/\sqrt{n}$ (page 297). Draw a solid **center line** on the chart at height μ.

center line

LOOK BACK

68–95–99.7 rule, p. 57

3. The 99.7 part of the 68–95–99.7 rule for Normal distributions says that, as long as the process remains in control, 99.7% of the values of $\bar{x}$ will fall between $\mu - 3\sigma/\sqrt{n}$ and $\mu + 3\sigma/\sqrt{n}$. Draw dashed control limits on the chart at these heights. The control limits mark off the range of variation in sample means that we expect to see when the process remains in control.

control limits

If the process remains in control and the process mean and standard deviation do not change, we will rarely observe an $\bar{x}$ outside the **control limits**. Such an $\bar{x}$ would be a signal that the process has been disturbed.

EXAMPLE 17.5

George Frey/Bloomberg via Getty Images

H2ORES

Monitoring the water resistance of fabric. A manufacturer of outdoor sportswear must control the water resistance and breathability of its jackets. Water resistance is measured by the amount of water (depth in millimeters) that can be suspended above the fabric before water seeps through. For its jackets, this test is done along the seams and zipper, where the resistance is likely the weakest. For one particular style of jacket, the manufacturing process has been stable with mean resistance $\mu = 2750$ mm and process standard deviation $\sigma = 430$ mm.

Each four-hour shift, an operator measures the resistance on a sample of four jackets. Table 17.1 gives the last 20 samples. The table also gives the mean $\bar{x}$ and the standard deviation s for each sample. The operator did not have to calculate these—modern measuring equipment often comes equipped with software that automatically records $\bar{x}$ and s and even produces control charts.

Figure 17.4 is an $\bar{x}$ control chart for the 20 water resistance samples in Table 17.1. We have plotted each sample mean from the table against its sample number. For example, the mean of the first sample is 2534 mm, and

TABLE 17.1 Twenty Control Chart Samples of Water Resistance (depth in mm)

Sample	Depth measurements				Sample mean	Standard deviation
1	2345	2723	2345	2723	2534	218
2	3111	3058	2385	2862	2854	330
3	2471	2053	2526	3161	2553	457
4	2154	2968	2742	2568	2608	344
5	3279	2472	2833	2326	2728	425
6	3043	2363	2018	2385	2452	428
7	2689	2762	2756	2402	2652	170
8	2821	2477	2598	2728	2656	150
9	2608	2599	2479	3453	2785	449
10	3293	2318	3072	2734	2854	425
11	2664	2497	2315	2652	2532	163
12	1688	3309	3336	3183	2879	797
13	3499	3342	2923	3015	3195	271
14	2352	2831	2459	2631	2568	210
15	2573	2184	2962	2752	2618	330
16	2351	2527	3006	2976	2715	327
17	2863	2938	2362	2753	2729	256
18	3281	2726	3297	2601	2976	365
19	3164	2874	3730	2860	3157	407
20	2968	3505	2806	2598	2969	388

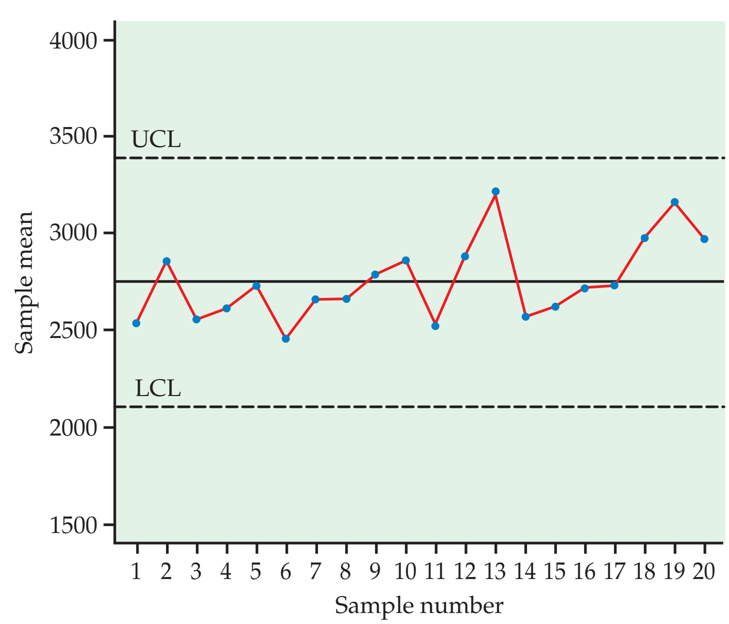

FIGURE 17.4 The $\bar{x}$ chart for the water resistance data of Table 17.1. No points lie outside the control limits.

this is the value plotted for Sample 1. The center line is at $\mu = 2750$ mm. The upper and lower control limits are

$$\mu + 3\frac{\sigma}{\sqrt{n}} = 2750 + 3\frac{430}{\sqrt{4}} = 2750 + 645 = 3395 \text{ mm} \quad \text{(UCL)}$$

$$\mu - 3\frac{\sigma}{\sqrt{n}} = 2750 - 3\frac{430}{\sqrt{4}} = 2750 - 645 = 2105 \text{ mm} \quad \text{(LCL)}$$

As is common, we have labeled the control limits UCL for upper control limit and LCL for lower control limit.

EXAMPLE 17.6

Reading an $\bar{x}$ control chart. Figure 17.4 is a typical $\bar{x}$ chart for a process in control. The means of the 20 samples do vary, but all lie within the range of variation marked out by the control limits. We are seeing the common cause variation of a stable process.

Figures 17.5 and 17.6 illustrate two ways in which the process can go out of control. In Figure 17.5, the process was disturbed by a special cause some time between Sample 12 and Sample 13. As a result, the mean resistance for Sample 13 falls above the upper control limit. It is common practice to mark all out-of-control points with an "x" to call attention to them. A search for the cause begins as soon as we see a point out of control. Investigation finds that the seam sealer device has slipped, resulting in more sealer being applied. This is good for water resistance but harms the jacket's breathability. When the problem is corrected, Samples 14 to 20 are again in control.

Figure 17.6 shows the effect of a steady upward drift in the process center, starting at Sample 11. You see that some time elapses before $\bar{x}$ is out of control (Sample 18). The one-point-out rule works better for detecting sudden large disturbances than for detecting slow drifts in a process.

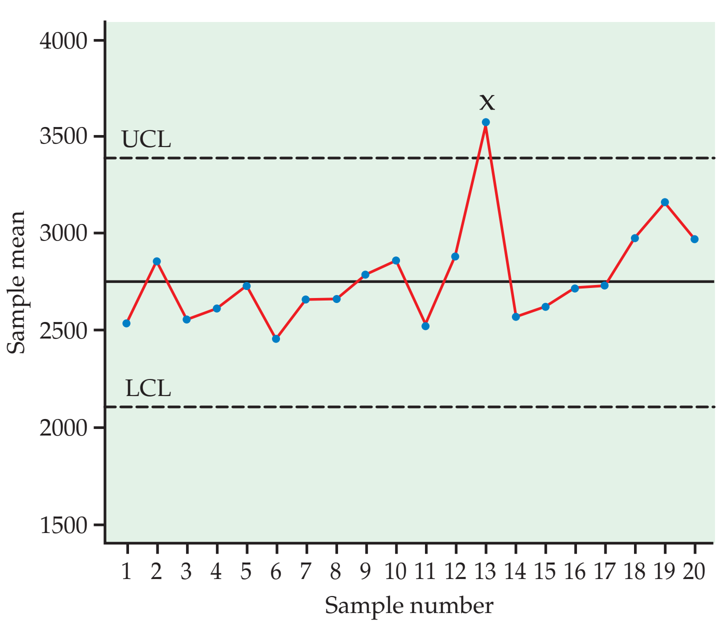

FIGURE 17.5 This $\bar{x}$ chart is identical to that in Figure 17.4 except that a special cause has driven $\bar{x}$ for Sample 13 above the upper control limit. The out-of-control point is marked with an x, Example 17.6.

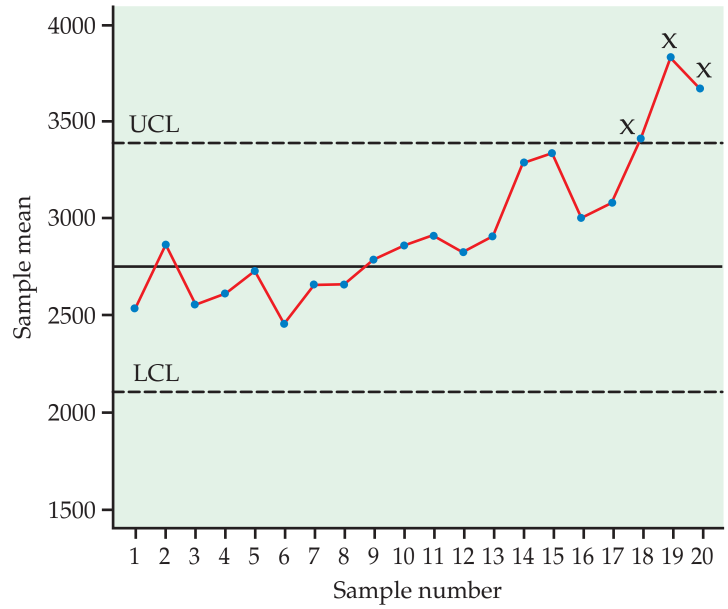

FIGURE 17.6 The first 10 points on this $\bar{x}$ chart are as in Figure 17.4. The process mean drifts upward after Sample 10, and the sample means $\bar{x}$ reflect this drift. The points for Samples 18, 19, and 20 are out of control, Example 17.6.

USE YOUR KNOWLEDGE

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17.5 An $\bar{x}$ control chart for sandwich orders. A sandwich shop owner takes a daily sample of five consecutive sandwich orders at a random time during the lunch rush and records the time it takes to complete each order. Past experience indicates that the process mean should be $\mu = 80$ seconds, and the process standard deviation should be $\sigma = 22$ seconds. Calculate the center line and control limits for an $\bar{x}$ control chart.

17.6 Changing the sample size n or the unit of measure. Refer to Exercise 17.5. What happens to the center line and control limits if

(a) The owner samples four consecutive sandwich orders?

(b) The owner samples six consecutive sandwich orders?

(c) The owner uses minutes rather than seconds as the units?

s charts for process monitoring

The $\bar{x}$ charts in Figures 17.4, 17.5, and 17.6 were easy to interpret because the process standard deviation remained fixed at 430 mm. The effects of moving the process mean away from its in-control value (2750 mm) are then clear to see. We know that even the simplest description of a distribution should give both a measure of center and a measure of spread. So it is with control charts. We must monitor both the process center, using an $\bar{x}$ chart, and the process spread, using a control chart for the sample standard deviation s.

The standard deviation s does not have a Normal distribution, even approximately. Under the process-monitoring conditions, the sampling distribution of s is skewed to the right. Nonetheless, control charts for any statistic are based on the "plus or minus 3 standard deviations" idea motivated by the 68–95–99.7 rule for Normal distributions.

Control charts are intended to be practical tools that are easy to use. Standard practice in process control therefore ignores such details as the effect of non-Normal sampling distributions. Here is the general control chart setup for a sample statistic Q (short for "quality characteristic").

THREE-SIGMA CONTROL CHARTS

To make a **three-sigma (3σ) control chart** for any statistic Q:

1. Take samples from the process at regular intervals and plot the values of the statistic Q against the order in which the samples were taken.
2. Draw a **center line** on the chart at height μ_Q, the mean of the statistic when the process is in control.
3. Draw upper and lower **control limits** on the chart three standard deviations of Q above and below the mean. That is,

$$\text{UCL} = \mu_Q + 3\sigma_Q$$
$$\text{LCL} = \mu_Q - 3\sigma_Q$$

   Here, σ_Q is the standard deviation of the sampling distribution of the statistic Q when the process is in control.
4. The chart produces an **out-of-control signal** when a plotted point lies outside the control limits.

We have applied this general idea to $\bar{x}$ charts. If μ and σ are the process mean and standard deviation, the statistic $\bar{x}$ has mean $\mu_{\bar{x}} = \mu$ and standard deviation $\sigma_{\bar{x}} = \sigma/\sqrt{n}$. The center line and control limits for $\bar{x}$ charts follow from these facts.

What are the corresponding facts for the sample standard deviation s? Study of the sampling distribution of s for samples from a Normally distributed process characteristic gives these facts:

1. The *mean* of s is a constant times the process standard deviation σ, that is, $\mu_s = c_4\sigma$.
2. The *standard deviation* of s is also a constant times the process standard deviation, $\sigma_s = c_5\sigma$.

The constants are called c_4 and c_5 for historical reasons. Their values depend on the size of the samples. For large samples, c_4 is close to 1. That is, the sample standard deviation s has little bias as an estimator of the process standard deviation σ. Because statistical process control often uses small samples, we pay attention to the value of c_4. Following the general pattern for three-sigma control charts,

1. The *center line* of an s chart is at $c_4\sigma$.
2. The *control limits* for an s chart are at

$$\text{UCL} = \mu_s + 3\sigma_s = c_4\sigma + 3c_5\sigma = (c_4 + 3c_5)\sigma = B_6\sigma$$
$$\text{LCL} = \mu_s - 3\sigma_s = c_4\sigma - 3c_5\sigma = (c_4 - 3c_5)\sigma = B_5\sigma$$

That is, the control limits UCL and LCL are also constants times the process standard deviation. These constants are called (again, for historical reasons) B_6 and B_5. We don't need to remember that $B_6 = c_4 + 3c_5$ and $B_5 = c_4 - 3c_5$, because tables give us the numerical values of B_6 and B_5.

$\bar{x}$ AND s CONTROL CHARTS FOR PROCESS MONITORING[5]

Take regular samples of size n from a process that has been in control with process mean μ and process standard deviation σ. The center line and control limits for an $\bar{x}$ **chart** are

$$\text{UCL} = \mu + 3\frac{\sigma}{\sqrt{n}}$$

$$\text{CL} = \mu$$

$$\text{LCL} = \mu - 3\frac{\sigma}{\sqrt{n}}$$

The center line and control limits for an s **chart** are

$$\text{UCL} = B_6\sigma$$
$$\text{CL} = c_4\sigma$$
$$\text{LCL} = B_5\sigma$$

The **control chart constants** c_4, B_5, and B_6 depend on the sample size n.

Table 17.2 gives the values of the control chart constants c_4, c_5, B_5, and B_6 for samples of sizes 2 to 10. This table makes it easy to draw s charts. The table has no B_5 entries for samples smaller than $n = 6$. The lower control limit for an s chart is zero for samples of sizes two to five. This is a consequence of the fact that s has a right-skewed distribution and takes only values greater than zero. The point 3 standard deviations above the mean (UCL) lies on the long right side of the distribution. The point 3 standard deviations below the mean (LCL) on the short left side is below zero, so we say that LCL = 0.

TABLE 17.2 Control Chart Constants

Sample size n	c_4	c_5	B_5	B_6
2	0.7979	0.6028		2.606
3	0.8862	0.4633		2.276
4	0.9213	0.3889		2.088
5	0.9400	0.3412		1.964
6	0.9515	0.3076	0.029	1.874
7	0.9594	0.2820	0.113	1.806
8	0.9650	0.2622	0.179	1.751
9	0.9693	0.2459	0.232	1.707
10	0.9727	0.2321	0.276	1.669

EXAMPLE 17.7

H2ORES

Interpreting an s chart for the waterproofing process. Figure 17.7 is the s chart for the water resistance data in Table 17.1. The samples are of size $n = 4$, and the process standard deviation in control is $\sigma = 430$ mm. The center line is, therefore,

$$\text{CL} = c_4\sigma = (0.9213)(430) = 396 \text{ mm}$$

The control limits are

$$\text{UCL} = B_6\sigma = (2.088)(430) = 898$$
$$\text{LCL} = B_5\sigma = (0)(430) = 0$$

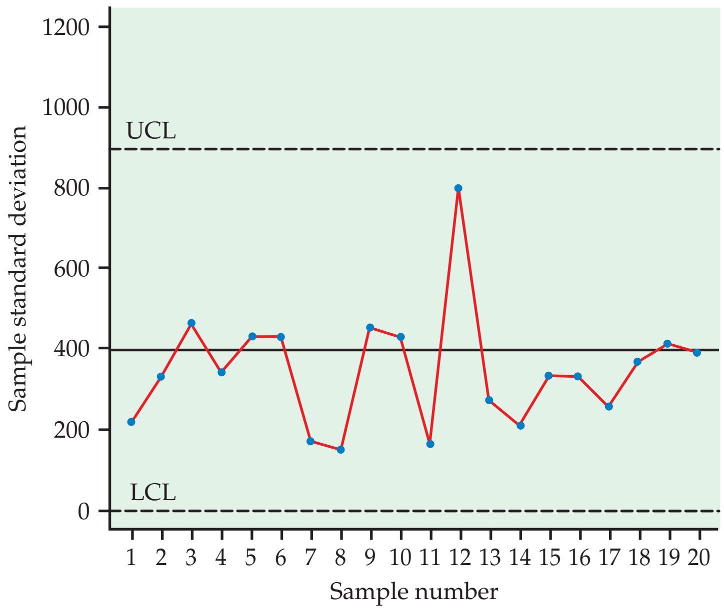


FIGURE 17.7 The s chart for the water resistance data of Table 17.1. Both the s chart and the $\bar{x}$ chart (Figure 17.4) are in control, Example 17.7.

Figures 17.4 and 17.7 go together: they are the $\bar{x}$ and s charts for monitoring the waterproofing process. Both charts are in control, showing only common cause variation within the bounds set by the control limits.

Figures 17.8 and 17.9 are $\bar{x}$ and s charts for the water resistance process when a new and poorly trained operator takes over the seam application between Samples 10 and 11. The new operator introduces added variation into the process, increasing the process standard deviation from its in-control value of 430 mm to 600 mm. The $\bar{x}$ chart in Figure 17.8 shows one point out of control. Only on closer inspection do we see that the spread of the $\bar{x}$'s increases after Sample 10. In fact, the process mean has remained unchanged at 2750 mm. The apparent lack of control in the $\bar{x}$ chart is entirely due to the larger process variation. There is a lesson here: *it is difficult to interpret an $\bar{x}$ chart unless s is in control. When you look at $\bar{x}$ and s charts, always start with the s chart.*

The s chart in Figure 17.9 shows lack of control starting at Sample 11. As usual, we mark the out-of-control points by an "x." The points for Samples

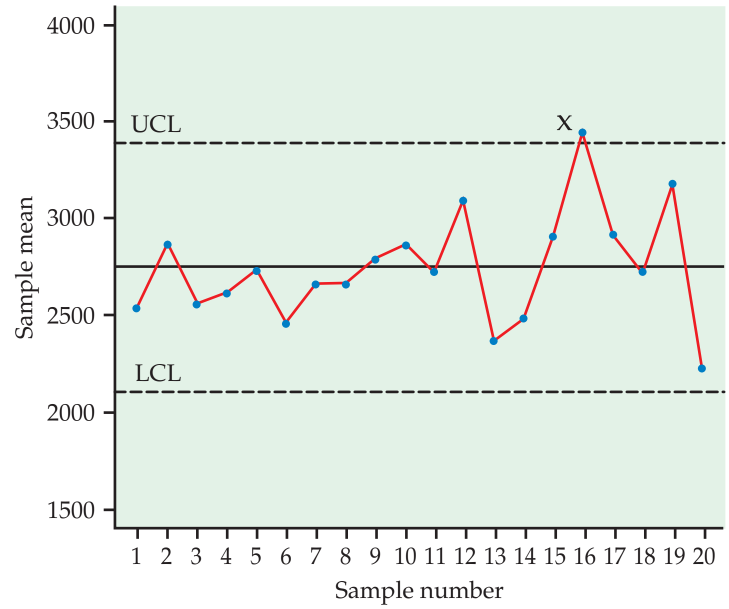


FIGURE 17.8 The $\bar{x}$ chart for water resistance when the process variability increases after Sample 10. The $\bar{x}$ chart does show the increased variability, but the s chart is clearer and should be read first, Example 17.7.

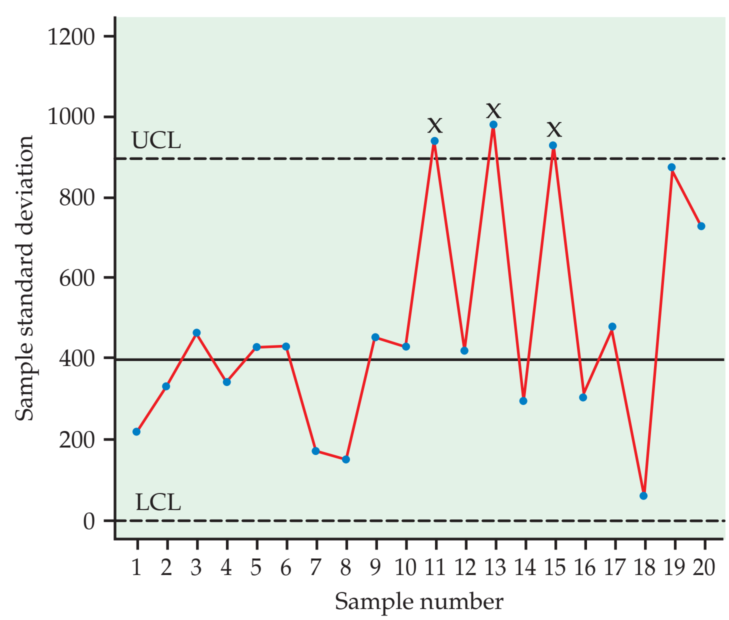


FIGURE 17.9 The s chart for water resistance when the process variability increases after Sample 10. Increased within-sample variability is clearly visible. Find and remove the s-type special cause before reading the $\bar{x}$ chart, Example 17.7.

13 and 15 also lie above the UCL, and the overall spread of the sample points is much greater than for the first 10 samples. In practice, the s chart would call for action after Sample 11. We would ignore the $\bar{x}$ chart until the special cause (the new operator) for the lack of control in the s chart has been found and removed by training the operator.

Example 17.7 suggests a strategy for using $\bar{x}$ and s charts in practice. First examine the s chart. Lack of control on an s chart is due to special causes that affect the observations *within a sample* differently. New and nonuniform raw material, a new and poorly trained operator, and mixing results from several machines or several operators are typical "s-type" special causes.

Once the s chart is in control, the stable value of the process standard deviation σ means that the variation within samples serves as a benchmark for detecting variation in the level of the process over the longer time periods between samples. The $\bar{x}$ chart, with control limits that depend on σ, does this. The $\bar{x}$ chart, as we saw in Example 17.7, responds to s-type causes as well as to longer-range changes in the process, so it is important to eliminate s-type special causes first. Then the $\bar{x}$ chart will alert us to, for example, a change in process level caused by new raw material that differs from that used in the past or a gradual drift in the process level caused by wear in a cutting tool.

EXAMPLE 17.8

Special causes and their effect on control charts. A large health maintenance organization (HMO) uses control charts to monitor the process of directing patient calls to the proper department or doctor's receptionist. Each day at a random time, five consecutive calls are recorded electronically. The first call today is handled quickly by an experienced operator, but the next goes to a newly hired operator who must ask a supervisor for help. The sample has a large s, and lack of control signals the need to train new hires more thoroughly.

The same HMO monitors the time required to receive orders from its main supplier of pharmaceutical products. After a long period in control, the $\bar{x}$ chart shows a systematic shift downward in the mean time because the supplier has changed to a more efficient delivery service. This is a desirable special cause, but nonetheless, it is a systematic change in the process. The HMO will have to establish new control limits that describe the new state of the process, with smaller process mean μ.

The second setting in Example 17.8 reminds us that a major change in the process returns us to the chart setup stage. In the absence of deliberate changes in the process, process monitoring uses the same values of μ and σ for long periods of time. One exception is common: careful monitoring and removal of special causes as they occur can permanently reduce the process σ. If the points on the s chart remain near the center line for a long period, it is wise to update the value of σ to the new, smaller value.

SECTION 17.1 SUMMARY

- Work is organized in **processes**, chains of activities that lead to some result. We use **flowcharts** and **cause-and-effect diagrams** to describe processes.

- All processes have variation. If the pattern of variation is stable over time, the process is **in statistical control**. **Control charts** are statistical plots intended to warn when a process is **out of control**.

- Standard 3σ **control charts** plot the values of some statistic Q for regular samples from the process against the time order of the samples. The **center line** is at the mean of Q. The **control limits** lie 3 standard deviations of Q above and below the center line. A point outside the control limits is an

out-of-control signal. For **process monitoring** of a process that has been in control, the mean and standard deviation are based on past data from the process and are updated regularly.

- When we measure some quantitative characteristic of the process, we use **$\bar{x}$ and s charts** for process control. The s chart monitors variation within individual samples. If the s chart is in control, the $\bar{x}$ chart monitors variation from sample to sample. To interpret the charts, always look first at the s chart.

SECTION 17.1 EXERCISES

For Exercises 17.1 and 17.2, see page 17-6; for Exercises 17.3 and 17.4, see page 17-8; and for Exercises 17.5 and 17.6, see page 17-12.

17.7 Constructing a flowchart. Consider the process of ordering a Jimmy John's sandwich order for delivery to your residence. Make a flowchart of this process, making sure to include steps that involve Yes/No decisions.

17.8 Determining sources of common and special cause variation. Refer to the previous exercise. The time it takes from deciding to order a sandwich to receiving the sandwich will vary. List several common causes of variation in this time. Then list several special causes that might result in unusual variation.

17.9 Constructing a Pareto chart. Comparisons are easier if you order the bars in a bar graph by height. A bar graph ordered from tallest to shortest bar is sometimes called a **Pareto chart**, after the Italian economist who recommended this procedure. Pareto charts are often used in quality studies to isolate the "vital few" categories on which we should focus our attention. Here is an example. Painting new auto bodies is a multistep process. There is an "electrocoat" that resists corrosion, a primer, a color coat, and a gloss coat. A quality study for one paint shop produced this breakdown of the primary problem type for those autos whose paint did not meet the manufacturer's standards:

Problem	Percent
Electrocoat uneven—redone	4
Poor adherence of color to primer	5
Lack of clarity in color	2
"Orange peel" texture in color	32
"Orange peel" texture in gloss	1
Ripples in color coat	28
Ripples in gloss coat	4
Uneven color thickness	19
Uneven gloss thickness	5
Total	100

Make a Pareto chart. Which stage of the painting process should we look at first?

17.10 Constructing another Pareto chart. A large hospital finds that it is losing money on surgery due to inadequate reimbursement by insurance companies and government programs. An initial study looks at losses broken down by diagnosis. Government standards place cases into diagnostic-related groups (DRGs). For example, major joint replacements are DRG 209. Here is what the hospital finds:

DRG	Percent of losses
104	5.2
107	10.1
109	7.7
116	13.7
148	6.8
209	15.2
403	5.6
430	6.8
462	9.4

What percent of total losses do these nine DRGs account for? Make a Pareto chart of losses by DRG. Which DRGs should the hospital study first when attempting to reduce its losses?

17.11 Making a Pareto chart. Continue the study of the process of calling in a sandwich order (Exercise 17.7). If you kept good records, you could make a Pareto chart of the reasons (special causes) for unusually long order times. Make a Pareto chart of these reasons. That is, list the reasons based on your experience and chart your estimates of the percent each reason explains.

17.12 Control limits for label placement. A rum producer monitors the position of its label on the bottle by sampling four bottles from each batch. One quantity measured is the distance from the bottom of the bottle neck to the top of the label. The process mean should be μ =1.8 inches. Past experience indicates that the distance varies with σ = 0.15 inch.

(a) The mean distance $\bar{x}$ for each batch sample is plotted on an $\bar{x}$ control chart. Calculate the center line and control limits for this chart.

(b) The sample standard deviation s for each batch's sample is plotted on an s control chart. What are the center line and control limits for this chart?

17.13 More on control limits for label placement. Refer to the previous exercise. What happens to the center line and control limits for the $\bar{x}$ and s control charts if

(a) The distributor samples 10 bottles from each batch?

(b) The distributor samples two bottles from each batch?

(c) The distributor uses centimeters rather than inches as the units?

17.14 Control limits for air conditioner thermostats. A maker of auto air conditioners checks a sample of five thermostatic controls from each hour's production. The thermostats are set at 72°F and then placed in a chamber where the temperature is raised gradually. The temperature at which the thermostat turns on the air conditioner is recorded. The process mean should be μ = 72°F. Past experience indicates that the response temperature of properly adjusted thermostats varies with s = 0.5°F.

(a) The mean response temperature $\bar{x}$ for each hour's sample is plotted on an $\bar{x}$ control chart. Calculate the center line and control limits for this chart.

(b) The sample standard deviation s for each hour's sample is plotted on an s control chart. What are the center line and control limits for this chart?

17.15 Control limits for a meat-packaging process. A meat-packaging company produces one-pound packages of ground beef by having a machine slice a long circular cylinder of ground beef as it passes through the machine. The timing between consecutive cuts will alter the weight of each section. Table 17.3 gives the weight of three consecutive sections of ground beef taken each hour over two 10-hour days. Past experience indicates that the process mean is 1.014 pounds and the weight varies with σ = 0.019 pound. MEATWGT

(a) Calculate the center line and control limits for an $\bar{x}$ chart.

(b) What are the center line and control limits for an s chart for this process?

(c) Create the $\bar{x}$ and s charts for these 20 consecutive samples.

(d) Does the process appear to be in control? Explain.

17.16 Causes of variation in the time to respond to an application. The personnel department of a large company records a number of performance

TABLE 17.3 Twenty Samples of Size 3, with $\bar{x}$ and s

Sample	Weight (pounds)			$\bar{x}$	s
1	0.999	1.071	1.019	1.030	0.0373
2	1.030	1.057	1.040	1.043	0.0137
3	1.024	1.020	1.041	1.028	0.0108
4	1.005	1.026	1.039	1.023	0.0172
5	1.031	0.995	1.005	1.010	0.0185
6	1.020	1.009	1.059	1.029	0.0263
7	1.019	1.048	1.050	1.039	0.0176
8	1.005	1.003	1.047	1.018	0.0247
9	1.019	1.034	1.051	1.035	0.0159
10	1.045	1.060	1.041	1.049	0.0098
11	1.007	1.046	1.014	1.022	0.0207
12	1.058	1.038	1.057	1.051	0.0112
13	1.006	1.056	1.056	1.039	0.0289
14	1.036	1.026	1.028	1.030	0.0056
15	1.044	0.986	1.058	1.029	0.0382
16	1.019	1.003	1.057	1.026	0.0279
17	1.023	0.998	1.054	1.025	0.0281
18	0.992	1.000	1.067	1.020	0.0414
19	1.029	1.064	0.995	1.029	0.0344
20	1.008	1.040	1.021	1.023	0.0159

measures. Among them is the time required to respond to an application for employment, measured from the time the application arrives. Suggest some plausible examples of each of the following.

(a) Reasons for common cause variation in response time.

(b) s-type special causes.

(c) $\bar{x}$-type special causes.

17.17 Control charts for a tablet compression process. A pharmaceutical manufacturer forms tablets by compressing a granular material that contains the active ingredient and various fillers. The hardness of a sample from each lot of tablets is measured in order to control the compression process. The process has been operating in control with mean at the target value $\mu = 11.5$ kiloponds (kp) and estimated standard deviation $\sigma = 0.2$ kp. Table 17.4 gives three sets of data, each representing $\bar{x}$ for 20 successive samples of $n = 4$ tablets. One set of data remains in control at the target value. In a second set, the process mean μ shifts suddenly to a new value. In a third, the process mean drifts gradually. PILL

(a) What are the center line and control limits for an $\bar{x}$ chart for this process?

(b) Draw a separate $\bar{x}$ chart for each of the three data sets. Mark any points that are beyond the control limits.

(c) Based on your work in part (b) and the appearance of the control charts, which set of data comes from a process that is in control? In which case does the process mean shift suddenly, and at about which sample do you think that the mean changed? Finally, in which case does the mean drift gradually?

17.18 More on the tablet compression process. Exercise 17.17 concerns process control data on the hardness of tablets for a pharmaceutical product. Table 17.5 gives data for 20 new samples of size 4, with the $\bar{x}$ and s for each sample. The process has been in control with mean at the target value $\mu = 11.5$ kp and standard deviation $\sigma = 0.2$ kp. PILL1

(a) Make both $\bar{x}$ and s charts for these data based on the information given about the process.

(b) At some point, the within-sample process variation increased from $\sigma = 0.2$ to $\sigma = 0.4$. About where in the 20 samples did this happen? What is the effect on the s chart? On the $\bar{x}$ chart?

(c) At that same point, the process mean changed from $\mu = 11.5$ to $\mu = 11.7$. What is the effect of this change on the s chart? On the $\bar{x}$ chart?

17.19 Control limits for a milling process. The width of a slot cut by a milling machine is important to the proper functioning of a hydraulic system for large

TABLE 17.4 Three Sets of $\bar{x}$'s from 20 Samples of Size 4

Sample	Data set A	Data set B	Data set C
1	11.602	11.627	11.495
2	11.547	11.613	11.475
3	11.312	11.493	11.465
4	11.449	11.602	11.497
5	11.401	11.360	11.573
6	11.608	11.374	11.563
7	11.471	11.592	11.321
8	11.453	11.458	11.533
9	11.446	11.552	11.486
10	11.522	11.463	11.502
11	11.664	11.383	11.534
12	11.823	11.715	11.624
13	11.629	11.485	11.629
14	11.602	11.509	11.575
15	11.756	11.429	11.730
16	11.707	11.477	11.680
17	11.612	11.570	11.729
18	11.628	11.623	11.704
19	11.603	11.472	12.052
20	11.816	11.531	11.905

TABLE 17.5 Twenty Samples of Size 4, with $\bar{x}$ and s

Sample	Hardness (kp)				$\bar{x}$	s
1	11.193	11.915	11.391	11.500	11.500	0.3047
2	11.772	11.604	11.442	11.403	11.555	0.1688
3	11.606	11.253	11.458	11.594	11.478	0.1642
4	11.509	11.151	11.249	11.398	11.326	0.1585
5	11.289	11.789	11.385	11.677	11.535	0.2362
6	11.703	11.251	11.231	11.669	11.463	0.2573
7	11.085	12.530	11.482	11.699	11.699	0.6094
8	12.244	11.908	11.584	11.505	11.810	0.3376
9	11.912	11.206	11.615	11.887	11.655	0.3284
10	11.717	11.001	11.197	11.496	11.353	0.3170
11	11.279	12.278	11.471	12.055	11.771	0.4725
12	12.106	11.203	11.162	12.037	11.627	0.5145
13	11.490	11.783	12.125	12.010	11.852	0.2801
14	12.299	11.924	11.235	12.014	11.868	0.4513
15	11.380	12.253	11.861	12.242	11.934	0.4118
16	11.220	12.226	12.216	11.824	11.872	0.4726
17	11.611	11.658	11.977	10.813	11.515	0.4952
18	12.251	11.481	11.156	12.243	11.783	0.5522
19	11.559	11.065	12.186	10.933	11.435	0.5681
20	11.106	12.444	11.682	12.378	11.902	0.6331

tractors. The manufacturer checks the control of the milling process by measuring a sample of six consecutive items during each hour's production. The target width for the slot is $\mu = 0.850$ inch. The process has been operating in control with center close to the target and $\sigma = 0.002$ inch. What center line and control limits should be drawn on the s chart? On the $\bar{x}$ chart?

17.20 Control limits for a dyeing process. The unique colors of the cashmere sweaters your firm makes result from heating undyed yarn in a kettle with a dye liquor. The pH (acidity) of the liquor is critical for regulating dye uptake and, hence, the final color. There are five kettles, all of which receive dye liquor from a common source. Twice each day, the pH of the liquor in each kettle is measured, giving a sample of size 5. The process has been operating in control with $\mu = 4.24$ and $\sigma = 0.137$.

(a) Give the center line and control limits for the s chart.

(b) Give the center line and control limits for the $\bar{x}$ chart.

17.21 Control charts for a mounting-hole process. Figure 17.10 reproduces a data sheet from a factory that makes electrical meters.[6] The sheet shows measurements of the distance between two mounting holes for 18 samples of size 5. The heading informs us that the measurements are in multiples of 0.0001 inch above 0.6000 inch. That is, the first measurement, 44, stands for 0.6044 inch. All the measurements end in 4. Although we don't know why this is true, it is clear that in effect the measurements were made to the nearest 0.001 inch, not to the nearest 0.0001 inch. Based on long experience with this process, you are keeping control charts based on $\mu = 43$ and $\sigma = 12.74$. Make s and $\bar{x}$ charts for the data in Figure 17.10 and describe the state of the process. MOUNT

17.22 Identifying special causes on control charts. The process described in Exercise 17.20 goes out of control. Investigation finds that a new type of yarn was recently introduced. The pH in the kettles is influenced by both the dye liquor and the yarn. Moreover, on a few occasions a faulty valve on one of the kettles had allowed water to enter that kettle; as a result, the yarn in that kettle had to be discarded. Which of these special causes appears on the s chart and which on the $\bar{x}$ chart? Explain your answer.

17.23 Determining the probability of detection. An $\bar{x}$ chart plots the means of samples of size 4 against center line CL = 715 and control limits LCL = 680 and UCL = 750. The process has been in control.

(a) What are the process mean and standard deviation?

(b) The process is disrupted in a way that changes the mean to $\mu = 700$. What is the probability that the first sample after the disruption gives a point beyond the control limits of the $\bar{x}$ chart?

(c) The process is disrupted in a way that changes the mean to $\mu = 700$ and the standard deviation to $\sigma = 10$. What is

Excel

VARIABLES CONTROL CHART ($\bar{x}$ & R)				Part No. 32506	Chart No. 1
Part name (project) Metal frame		Operation (process) Distance between mounting holes		Specification limits 0.6054" ± 0.0010"	
Operator	Machine R-5	Gage	Unit of measure 0.0001"	Zero equals 0.6000"	

Date	3/7				3/8						3/9							
Time	8:30	10:30	11:45	1:30	8:15	10:15	11:45	2:00	3:00	4:00	8:30	10:00	11:45	1:30	2:30	3:30	4:30	5:30
Sample measurements 1	44	64	34	44	34	34	54	64	24	34	34	54	44	24	54	54	54	54
2	44	44	44	54	14	64	64	34	54	44	44	44	24	24	24	34	34	24
3	44	34	54	54	84	34	34	54	44	44	34	24	34	54	54	24	74	64
4	44	34	44	34	54	44	44	44	34	34	64	54	34	44	44	44	44	34
5	64	54	54	44	44	44	34	44	34	34	34	24	44	44	44	54	54	44
Average, $\bar{x}$																		
Range, R	20	30	20	20	70	30	30	30	30	10	30	30	20	30	30	30	40	40

FIGURE 17.10 A process control record sheet kept by operators, Exercise 17.21. This is typical of records kept by hand when measurements are not automated. We will see in the next section why such records mention $\bar{x}$ and R control charts rather than $\bar{x}$ and s charts.

the probability that the first sample after the disruption gives a point beyond the control limits of the $\bar{x}$ chart?

17.24 Alternative control limits. American and Japanese practice uses 3σ control charts. That is, the control limits are three standard deviations on either side of the mean. When the statistic being plotted has a Normal distribution, the probability of a point outside the limits is about 0.003 (or about 0.0015 in each direction) by the 68–95–99.7 rule (page 57). European practice uses control limits placed so that the probability of a point outside the limits when in control is 0.001 in each direction. For a Normally distributed statistic, how many standard deviations on either side of the mean do these alternative control limits lie?

17.25 2σ control charts. Some special situations call for 2σ control charts. That is, the control limits for a statistic Q will be $\mu_Q \pm 2\sigma_Q$. Suppose that you know the process mean μ and standard deviation σ and will plot $\bar{x}$ and s from samples of size n.

(a) What are the 2σ control limits for an $\bar{x}$ chart?

(b) Find expressions for the upper and lower 2σ control limits for an s chart in terms of the control chart constants c_4 and c_5 introduced on page 17-14.

17.2 Using Control Charts

When you complete this section, you will be able to:

- Implement various out-of-control rules when interpreting control charts.
- Set up a control chart based on past data.
- Identify rational subgroups when deciding how to choose samples.
- Distinguish between the natural tolerances for a product and the control limits for a process, as well as between capability and control.

The previous section discussed the ideas behind control charts as well as the details of making $\bar{x}$ and s charts. This section discusses a variety of topics related to using control charts in practice.

$\bar{x}$ and R charts

sample range

We have seen that it is essential to monitor both the center and the spread of a process. Control charts were originally intended to be used by factory workers with limited knowledge of statistics in the era before even calculators, let alone software, were common. In that environment, the standard deviation is too difficult to calculate. Thus, the $\bar{x}$ chart for center was used with a control chart for spread based on the **sample range** rather than the sample standard deviation.

R chart

The range R of a sample is just the difference between the largest and smallest observations. It is easy to find R without a calculator. Using R rather than s to measure the spread of samples replaces the s chart with an ***R* chart**. It also changes the $\bar{x}$ chart because the control limits for $\bar{x}$ use the estimated process spread.

Because the range R uses only the largest and smallest observations in a sample, it is less informative than the standard deviation s calculated from all the observations. For this reason, $\bar{x}$ and s charts are now preferred to $\bar{x}$ and R charts. R charts, however, remain common because it is easier for workers to understand R than s.

In this short introduction, we concentrate on the principles of control charts, so we won't give the details of constructing $\bar{x}$ and R charts. These details appear in any text on quality control.[7] If you meet a set of $\bar{x}$ and R charts, remember that the interpretation of these charts is just like the interpretation of $\bar{x}$ and s charts.

EXAMPLE 17.9

Example of a typical process control technology. Figure 17.11 is a display produced by custom process control software attached to a laser micrometer. In this demonstration prepared by the software maker, the micrometer

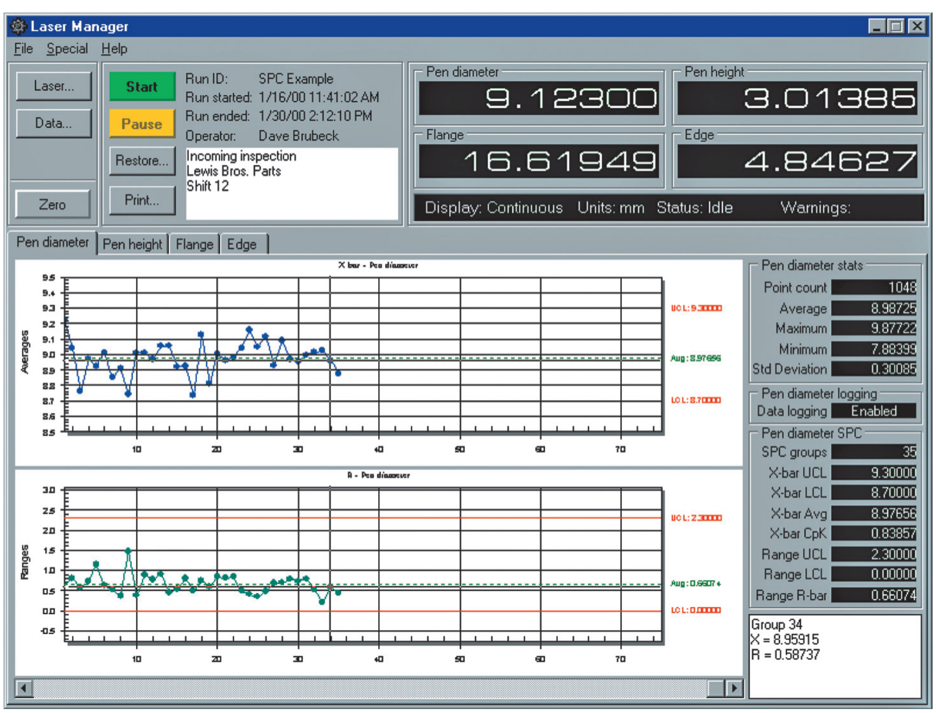


FIGURE 17.11 Output for operators, from the Laser Manager software by System Dynamics, Inc. The software prepares control charts directly from measurements made by a laser micrometer, Example 17.9. Compare the hand record sheet in Figure 17.10. (Image provided by Gordon A. Feingold, System Dynamics, Inc. Used by permission.)

is measuring the diameter in millimeters of samples of pens shipped by an office supply company. The software controls the laser, records measurements, makes the control charts, and sounds an alarm when a point is out of control. This is typical of process control technology in modern manufacturing settings.

The software presents $\bar{x}$ and R charts rather than $\bar{x}$ and s charts. The R chart monitors within-sample variation (just like an s chart), so we look at it first. We see that the process spread is stable and well within the control limits. Just as in the case of s, the LCL for R is 0 for the samples of size $n = 5$ used here. The $\bar{x}$ chart is also in control, so process monitoring will continue. The software will sound an alarm if either chart goes out of control.

USE YOUR KNOWLEDGE

17.26 What's wrong? For each of the following, explain what is wrong and why.

(a) The R chart monitors the center of the process.

(b) The R chart is commonly used because the range R is more informative than the standard deviation s.

(c) Use of the range R to monitor process spread does not alter the construction of the control limits for the $\bar{x}$ chart.

Additional out-of-control rules

So far, we have used only the basic "one point beyond the control limits" criterion to signal that a process may have gone out of control. We would like a quick signal when the process moves out of control, but we also want to avoid "false alarms," signals that occur just by chance when the process is really in control.

The standard 3σ control limits are chosen to prevent too many false alarms because an out-of-control signal calls for an effort to find and remove a special cause. As a result, $\bar{x}$ charts are often slow to respond to a gradual drift in the process center.

We can speed the response of a control chart to lack of control—at the cost of also enduring more false alarms—by adding patterns other than "one-point-out" as rules. The most common step in this direction is to add a *runs rule* to the $\bar{x}$ chart.

OUT-OF-CONTROL SIGNALS

$\bar{x}$ and s or $\bar{x}$ and R control charts produce an out-of-control signal if

1. **One-point-out:** A single point lies outside the 3σ control limits of either chart.
2. **Run:** The $\bar{x}$ chart shows nine consecutive points above the center line or nine consecutive points below the center line. The signal occurs when we see the ninth point of the run.

EXAMPLE 17.10

Effectiveness of the runs rule. Figure 17.12 reproduces the $\bar{x}$ chart from Figure 17.6. The process center began a gradual upward drift at Sample 11. The chart shows the effect of the drift—the sample means plotted on the chart move gradually upward, with some random variation. The one-point-out rule does not call for action until Sample 18 finally produces an $\bar{x}$ above the UCL. The runs rule reacts slightly more quickly: Sample 17 is the ninth consecutive point above the center line.

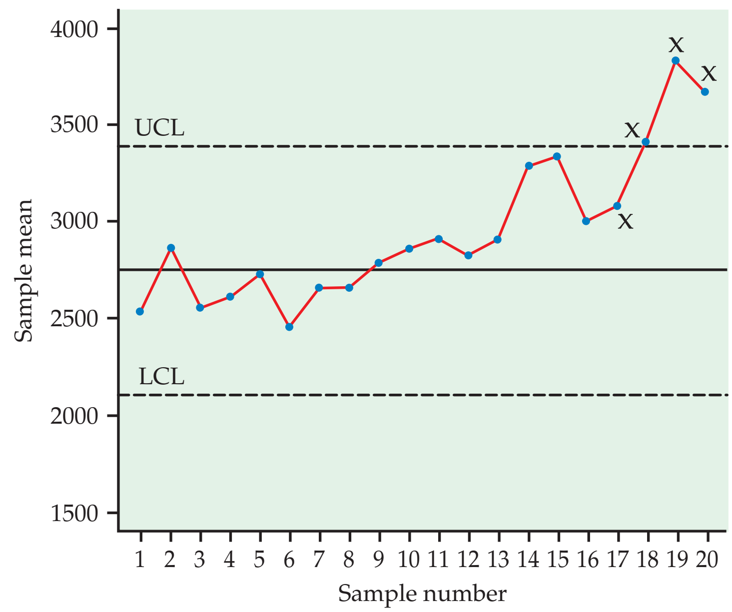


FIGURE 17.12 The $\bar{x}$ chart for water resistance data when the process center drifts upward, Example 17.10. The "run of 9" signal gives an out-of-control warning at Sample 17.

It is a mathematical fact that the runs rule responds to a gradual drift more quickly (on the average) than the one-point-out rule does. The motivation for a runs rule is that when a process is in control, half the points on an $\bar{x}$ chart should lie above the center line and half below. That's true, on the average, in the long term. In the short term, we will see runs of points above or below, just as we see runs of heads or tails in tossing a coin.

To determine how long a run must be to suggest that the process center has moved, we once again concern ourselves with the cost of false alarms. The 99.7 part of the 68–95–99.7 rule says that we will get a point outside the 3σ control limits about three times for every 1000 points plotted when the process is in control. The chance of nine straight points above the center line when the process is in control is $(1/2)^9 = 1/512$, or about two per 1000. The chance for a run of nine below the center line is the same. Combined, that's about four false alarms per 1000 plotted points overall when the process is in control. This is very close to the false-alarm rate for one-point-out.

There are many other patterns that can be added to the rules for responding to $\bar{x}$ and s or $\bar{x}$ and R charts. In our enthusiasm to detect various special kinds of loss of control, it is easy to forget that adding rules always increases the frequency of false alarms. Frequent false alarms are so annoying that the people responsible for responding soon begin to ignore out-of-control signals. *It is better to use only a few out-of-control rules and to reserve rules other than one-point-out and runs for processes that are known to be prone to specific special causes for which there are tailor-made detection rules.*[8]

USE YOUR KNOWLEDGE

17.27 What's wrong? For each of the following, explain what is wrong and why.

(a) For the one-point-out rule, you could reduce the frequency of false alarms by using 2σ control limits.

(b) In speeding up the response of a control chart to lack of control, we decrease the frequency of false alarms.

(c) The runs rule is designed to quickly detect a large and sudden shift in the process.

17.28 The effect of special cause variation. Is each of the following examples of a special cause most likely to first result in (i) one-point-out on the s or R chart, (ii) one-point-out on the $\bar{x}$ chart, or (iii) a run on the $\bar{x}$ chart? In each case, briefly explain your reasoning.

(a) An etching solution deteriorates as more items are etched.

(b) Buildup of dirt reduces the precision with which parts are placed for machining.

(c) A new customer service representative for a Spanish-language help line is not a native speaker and has difficulty understanding customers.

(d) A data entry employee grows less attentive as his shift continues.

Setting up control charts

When you first encounter a process that has not been carefully studied, it is quite likely that the process is not in control. Your first goal is to discover and remove special causes and so bring the process into control. Control charts are an important tool. Control charts for *process monitoring* follow the process forward in time to keep it in control. Control charts at the *chart setup* stage, on the other hand, look back in an attempt to discover the present state of the process. An example will illustrate the method.

EXAMPLE 17.11

VISC

Monitoring the viscosity of a material. The viscosity of a material is its resistance to flow when under stress. Viscosity is a critical characteristic of rubber and rubber-like compounds called elastomers, which have many uses in consumer products. Viscosity is measured by placing specimens of the material above and below a slowly rotating roller, squeezing the assembly, and recording the drag on the roller. Measurements are in "Mooney units," named after the inventor of the instrument.

A specialty chemical company is beginning production of an elastomer that is supposed to have viscosity 45 ± 5 Mooneys. Each lot of the elastomer

TABLE 17.6 $\bar{x}$ **and** s **for 24 Samples of Elastomer Viscosity (in Mooneys)**

Sample	$\bar{x}$	s	Sample	$\bar{x}$	s
1	49.750	2.684	13	47.875	1.118
2	49.375	0.895	14	48.250	0.895
3	50.250	0.895	15	47.625	0.671
4	49.875	1.118	16	47.375	0.671
5	47.250	0.671	17	50.250	1.566
6	45.000	2.684	18	47.000	0.895
7	48.375	0.671	19	47.000	0.447
8	48.500	0.447	20	49.625	1.118
9	48.500	0.447	21	49.875	0.447
10	46.250	1.566	22	47.625	1.118
11	49.000	0.895	23	49.750	0.671
12	48.125	0.671	24	48.625	0.895

is produced by "cooking" raw material with catalysts in a reactor vessel. Table 17.6 records $\bar{x}$ and s from samples of size $n = 4$ lots from the first 24 shifts as production begins.[9] An s chart, therefore, monitors variation among lots produced during the same shift. If the s chart is in control, an $\bar{x}$ chart looks for shift-to-shift variation.

Estimating μ We do not know the process mean μ and standard deviation σ. What shall we do? Sometimes, we can easily adjust the center of a process by setting some control, such as the depth of a cutting tool in a machining operation or the temperature of a reactor vessel in a pharmaceutical plant. In such cases, it is common to simply take the process mean μ to be the target value, the depth or temperature that the design of the process specifies as correct. The $\bar{x}$ chart then helps us keep the process mean at this target value.

There is less likely to be a "correct value" for the process mean μ if we are monitoring response times to customer calls or data entry errors. In Example 17.11, we have the target value 45 Mooneys, but there is no simple way to set viscosity at the desired level. In such cases, we want the μ we use in our $\bar{x}$ chart to describe the center of the process as it has actually been operating. To do this, take the mean of all the individual measurements in the past samples. Because the samples are all the same size, this is just the mean of the sample $\bar{x}$'s. The overall "mean of the sample means" is therefore usually called $\bar{\bar{x}}$. For the 24 samples in Table 17.6,

$$\bar{\bar{x}} = \frac{1}{24}(49.750 + 49.375 + \cdots + 48.625)$$
$$= \frac{1161.125}{24} = 48.380$$

Estimating σ *It is almost never safe to use a "target value" for the process standard deviation σ because it is almost never possible to directly adjust process variation.* We must estimate σ from past data. We want to combine the sample standard deviations s from past samples rather than use the standard deviation of all the individual observations in those samples. That is, in Example 17.11, we want to combine the 24 sample standard deviations in Table 17.6 rather than calculate the standard deviation of the 96 observations in these samples. The reason is that it is the *within-sample* variation that is the benchmark against which we compare the longer-term process variation. Even if the process has been in control, we want only the variation over the short time period of a single sample to influence our value for σ.

There are several ways to estimate σ from the sample standard deviations. Software may use a somewhat sophisticated method and then calculate the control limits for you. Here, we use a simple method that is traditional in quality control because it goes back to the era before software. If we are basing chart setup on k past samples, we have k sample standard deviations $s_1, s_2, \ldots, s_k$. Just average these to get

$$\bar{s} = \frac{1}{k}(s_1 + s_2 + \cdots + s_k)$$

For the viscosity example, we average the s-values for the 24 samples in Table 17.6,

$$\bar{s} = \frac{1}{24}(2.684 + 0.895 + \cdots + 0.895)$$

$$= \frac{24.156}{24} = 1.0065$$

LOOK BACK
mean of s, p. 17-13

Combining the sample s-values to estimate σ introduces a complication: the samples used in process control are often small (size $n = 4$ in the viscosity example), so s has some bias as an estimator of σ. The estimator $\bar{s}$ inherits this bias. A proper estimate of σ corrects this bias. Thus, our estimator is

$$\hat{\sigma} = \frac{\bar{s}}{c_4}$$

We get control limits from past data by using the estimates $\bar{\bar{x}}$ and $\hat{\sigma}$ in place of the μ and σ used in charts at the process-monitoring stage. Here are the results.[10]

$\bar{x}$ AND s CONTROL CHARTS USING PAST DATA

Take regular samples of size n from a process. Estimate the process mean μ and the process standard deviation σ from past samples by

$$\hat{\mu} = \bar{\bar{x}} \qquad \text{(or use a target value)}$$

$$\hat{\sigma} = \frac{\bar{s}}{c_4}$$

The center line and control limits for an **$\bar{x}$ chart** are

$$\text{UCL} = \hat{\mu} + 3\frac{\hat{\sigma}}{\sqrt{n}}$$

$$\text{CL} = \hat{\mu}$$

$$\text{LCL} = \hat{\mu} - 3\frac{\hat{\sigma}}{\sqrt{n}}$$

The center line and control limits for an ***s* chart** are

$$\text{UCL} = B_6\hat{\sigma}$$
$$\text{CL} = c_4\hat{\sigma} = \bar{s}$$
$$\text{LCL} = B_5\hat{\sigma}$$

If the process was not in control when the samples were taken, these should be regarded as trial control limits.

Chart setup We are now ready to outline the chart setup procedure for the elastomer viscosity.

Step 1. As usual, we look first at an *s* chart. For chart setup, control limits are based on the same past data that we will plot on the chart. Based on Table 17.6,

$$\bar{s} = 1.0065$$
$$\hat{\sigma} = \frac{\bar{s}}{c_4} = \frac{1.0065}{0.9213} = 1.0925$$

So the center line and control limits for the *s* chart are

$$\text{UCL} = B_6\hat{\sigma} = (2.088)(1.0925) = 2.281$$
$$\text{CL} = \bar{s} = 1.0065$$
$$\text{LCL} = B_5\hat{\sigma} = (0)(1.0925) = 0$$

Figure 17.13 is the *s* chart. The points for Shifts 1 and 6 lie above the UCL. Both are near the beginning of production. Investigation finds that the reactor operator made an error on one lot in each of these samples. The error changed the viscosity of those two lots and increased *s* for each of the samples. The error will not be repeated now that the operators have gained experience. That is, this special cause has already been removed.

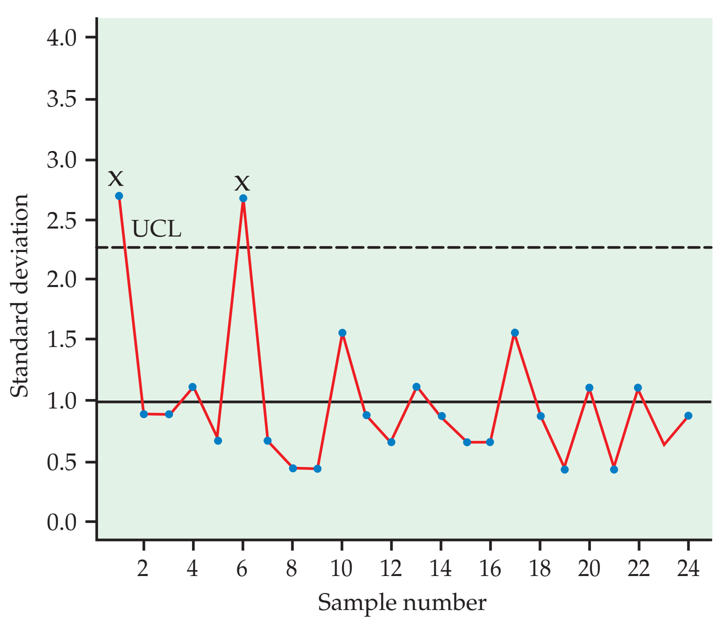


FIGURE 17.13 The *s* chart based on past data for the viscosity data of Table 17.6. The control limits are based on the same *s*-values that are plotted on the chart. Points 1 and 6 are out of control.

Step 2. Remove the two values of s that were out of control. This is proper because the special cause responsible for these readings is no longer present. From the remaining 22 shifts

$$\bar{s} = 0.854 \quad \text{and} \quad \hat{\sigma} = \frac{0.854}{0.9213} = 0.927$$

The new s chart center line and control limits are

$$\begin{aligned} \text{UCL} &= B_6\hat{\sigma} = (2.088)(0.927) = 1.936 \\ \text{CL} &= \bar{s} = 0.854 \\ \text{LCL} &= B_5\hat{\sigma} = (0)(0.927) = 0 \end{aligned}$$

We don't show this chart, but you can see from Table 17.6 and Figure 17.13 that none of the remaining s-values lies above the new, lower UCL; the largest remaining s is 1.566. If additional points were out of control, we would repeat the process of finding and eliminating s-type causes until the s chart for the remaining shifts is in control. In practice, this is often a challenging task.

Step 3. Once s-type causes have been eliminated, make an $\bar{x}$ chart *using only the samples that remain* after dropping those that had out-of-control s-values. For the 22 remaining samples, we calculate $\bar{\bar{x}} = 48.4716$ and we know that $\hat{\sigma} = 0.927$. The center line and control limits for the $\bar{x}$ chart are

$$\begin{aligned} \text{UCL} &= \bar{\bar{x}} + 3\frac{\hat{\sigma}}{\sqrt{n}} = 48.4716 + 3\frac{0.927}{\sqrt{4}} = 49.862 \\ \text{CL} &= \bar{\bar{x}} = 48.4716 \\ \text{LCL} &= \bar{\bar{x}} - 3\frac{\hat{\sigma}}{\sqrt{n}} = 48.4716 - 3\frac{0.927}{\sqrt{4}} = 47.081 \end{aligned}$$

Figure 17.14 is the $\bar{x}$ chart. Shifts 1 and 6 were already dropped. Seven of the remaining 22 points are beyond the 3σ limits, four high and three low. Although within-shift variation is now stable, there is excessive variation from

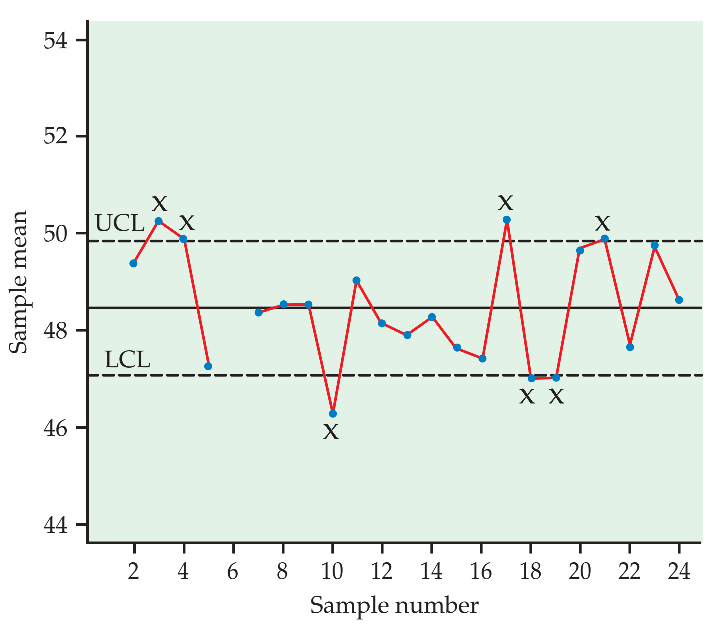


FIGURE 17.14 The $\bar{x}$ chart based on past data for the viscosity data of Table 17.6. The samples for Shifts 1 and 6 have been removed because s-type special causes active in those samples are no longer active. The $\bar{x}$ chart shows poor control.

shift to shift. To find the cause, we must understand the details of the process, but knowing that the special cause or causes operate between shifts is a big help. If the reactor is set up anew at the beginning of each shift, that's one place to look more closely.

Step 4. Once the $\bar{x}$ and s charts are both in control (looking backward), use the estimates $\hat{\mu}$ and $\hat{\sigma}$ from the points in control to set tentative control limits to monitor the process going forward. If it remains in control, we can update the charts and move to the process-monitoring stage.

USE YOUR KNOWLEDGE

MEATWGT

17.29 Updating control chart limits. Suppose that when the process improvement project of Example 17.11 (page 17-26) is complete, the points remaining after removing special causes have $\bar{\bar{x}} = 47.2$ and $\bar{s} = 1.03$. What are the center line and control limits for the $\bar{x}$ and s charts you would use to monitor the process going forward?

17.30 More on updating control chart limits. In Exercise 17.15, control limits for the weight of ground beef were obtained using historical results. Using Table 17.3 (page 17-19), estimate the process μ and process σ. Do either of these values suggest a change in the process center and spread?

Comments on statistical control

Having seen how $\bar{x}$ and s (or $\bar{x}$ and R) charts work, we can turn to some important comments and cautions about statistical control in practice.

Focus on the process rather than on the product This is perhaps the fundamental idea in statistical process control. We might attempt to attain high quality by careful inspection of the finished product and reviewing every outgoing invoice and expense account payment. Inspection of finished products can ensure good quality, but it is expensive.

Perhaps more important, final inspection often comes too late: when something goes wrong early in a process, a lot of bad product may be produced before final inspection discovers the problem. This adds to the expense because the bad product must then be scrapped or reworked.

The small samples that are the basis of control charts are intended to monitor the process at key points, not to ensure the quality of the particular items in the samples. If the process is kept in control, we know what to expect in the finished product. We want to do it right the first time, not inspect and fix finished product.

Choosing the "key points" at which we will measure and monitor the process is important. The choice requires that you understand the process well enough to know where problems are likely to arise. Flowcharts and cause-and-effect diagrams can help. It should be clear that control charts that monitor only the final output are often *not* the best choice.

Rational subgroups The interpretation of control charts depends on the distinction between $\bar{x}$-type special causes and s-type special causes. This distinction, in turn, depends on how we choose the samples from which we calculate s (or R). We want the variation *within* a sample to reflect only the item-to-item chance variation that (when in control) results from many small common causes. Walter Shewhart, the founder of statistical process control, used the term **rational subgroup** to emphasize that we should think about the process when deciding how to choose samples.

rational subgroup

EXAMPLE 17.12

Selecting the sample. A pharmaceutical manufacturer forms tablets by compressing a granular material that contains the active ingredient and various fillers. To monitor the compression process, we will measure the hardness of a sample from each 10 minutes' production of tablets. Should we choose a random sample of tablets from the several thousand produced in a 10-minute period?

A random sample would contain tablets spread across the entire 10 minutes. It fairly represents the 10-minute period, but that isn't what we want for process control. If the setting of the press drifts or a new lot of filler arrives during the 10 minutes, the spread of the sample will be increased. That is, a random sample contains both the short-term variation among tablets produced in quick succession and the longer-term variation among tablets produced minutes apart. We prefer to measure a rational subgroup of five consecutive tablets every 10 minutes. We expect the process to be stable during this very short time period, so that variation within the subgroups is a benchmark against which we can see special cause variation.

Samples of consecutive items are rational subgroups when we are monitoring the output of a single activity that does the same thing over and over again. *Several consecutive items is the most common type of sample for process control.*

When the stream of product contains output from several machines or several people, however, the choice of samples is more complicated. Do you want to include variation due to different machines or different people within your samples? If you decide that this variation is common cause variation, be sure that the sample items are spread across machines or people. If all the items in each sample have a common origin, $\bar{s}$ will be small and the control limits for the $\bar{x}$ chart will be narrow. Points on the $\bar{x}$ chart from samples representing different machines or different people will often be out of control, some high and some low.

CAUTION

There is no formula for deciding how to form rational subgroups. You must think about causes of variation in your process and decide which you are willing to think of as common causes that you will not try to eliminate. Rational subgroups are samples chosen to express variation due to these causes and no others. Because the choice requires detailed process knowledge, we will usually accept samples of consecutive items as being rational subgroups. Just remember that real processes are messier than textbooks suggest.

Why statistical control is desirable To repeat, if the process is kept in control, we know what to expect in the finished product. The process mean μ and standard deviation σ remain stable over time, so (assuming Normal variation) the 99.7 part of the 68–95–99.7 rule tells us that almost all measurements on individual products will lie in the range $\mu \pm 3\sigma$. These are sometimes called the **natural tolerances** for the product. Be careful to distinguish $\mu \pm 3\sigma$, the range we expect for *individual measurements*, from the $\bar{x}$ chart control limits $\mu \pm 3\sigma/\sqrt{n}$, which mark off the expected range of *sample means*.

natural tolerances

EXAMPLE 17.13

Estimating the tolerances for the water resistance study. The process of waterproofing the jackets has been operating in control. The $\bar{x}$ and s charts were based on $\mu = 2750$ mm and $\sigma = 430$ mm. The s chart in Figure 17.7 and a calculation (see Exercise 17.35, page 17-37) suggest that the process σ is now less than 430 mm. We may prefer to calculate the natural tolerances from the recent data on 20 samples (80 jackets) in Table 17.1. The estimate of the mean is $\bar{\bar{x}} = 2750.7$, very close to the target value.

Now a subtle point arises. The estimate $\hat{\sigma} = \bar{s}/c_4$ used for past-data control charts is based entirely on variation *within the samples*. That's what we want for control charts because within-sample variation is likely to be "pure common cause" variation.

Even when the process is in control, there is some additional variation from sample to sample, just by chance. So, the variation in the process output will be greater than the variation within samples. *To estimate the natural tolerances, we should estimate σ from all 80 individual jackets rather than by averaging the 20 within-sample standard deviations.* The standard deviation for all 80 jackets is

$$s = 383.8$$

For a sample of size 80, c_4 is very close to 1, so we can ignore it. We are, therefore, confident that almost all individual jackets will have a water resistance reading between

$$\bar{\bar{x}} \pm 3s = 2750.7 \pm (3)(383.8) \doteq 2750.7 \pm 1151.4$$

We expect water resistance measurements to vary between 1599 and 3902 mm. You see that the spread of individual measurements is wider than the spread of sample means used for the control limits of the $\bar{x}$ chart.

The natural tolerances in Example 17.13 depend on the fact that the water resistance of individual jackets follows a Normal distribution. We know that the process was in control when the 80 measurements in Table 17.1 were made, so we can use them to assess Normality. Figure 17.15 is a Normal

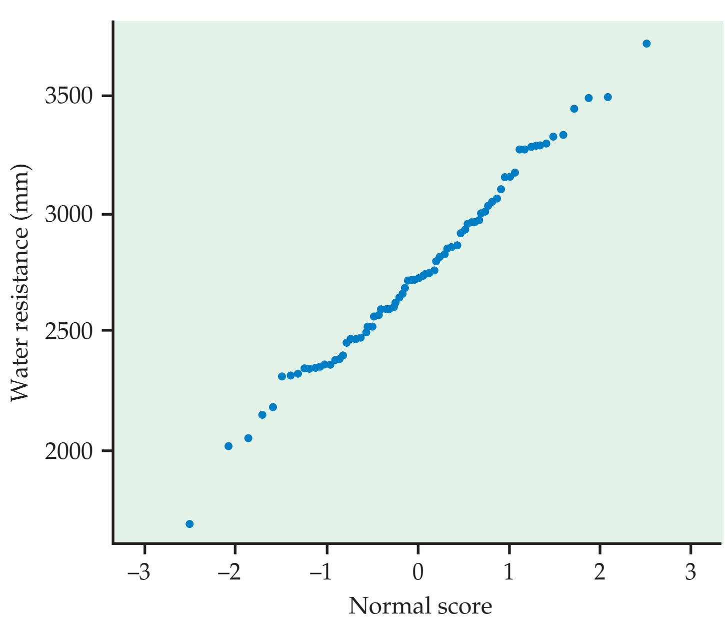


FIGURE 17.15 Normal quantile plot for the 80 water resistance measurements of Table 17.1. Calculations about individual measurements, such as natural tolerances, depend on approximate Normality.

quantile plot of these measurements. There are no strong deviations from Normality. All 80 observations, including the one point that may appear suspiciously low in Figure 17.15, lie within the natural tolerances. Examining the data strengthens our confidence in the natural tolerances.

Because we can predict the performance of the waterproofing process, we can tell the buyers of our jackets what to expect. What is more, if a process is in control, we can see the effect of any changes we make. A process operating out of control is erratic. We can't do reliable statistical studies on such a process, and if we make a change in the process, we can't clearly see the results of the change—they are hidden by erratic special cause variation. If we want to improve a process, we must first bring it into control so that we have a stable starting point from which to improve.

Don't confuse control with capability!

A process in control is stable over time, and we know how much variation the finished product will show. Control charts are, so to speak, the voice of the process telling us what state it is in. *There is no guarantee that a process in control produces products of satisfactory quality.* "Satisfactory quality" is measured by comparing the product to some standard outside the process set by technical specifications, customer expectations, or the goals of the organization. These external standards are unrelated to the internal state of the process, which is all that statistical control pays attention to.

CAPABILITY

Capability refers to the ability of a process to meet or exceed the requirements placed on it.

Capability has nothing to do with control—except for the very important point that if a process is not in control, it is hard to tell if it is capable or not.

EXAMPLE 17.14

Assessing the capability of the waterproofing process. An outfitting company is a large buyer of this jacket. They informed us that they need water resistance levels between 1000 and 4000 mm. Although the waterproofing process is in control, we know (Example 17.13) that almost all jackets will have water resistance levels between 1599 and 3902 mm. The process is capable of meeting the customer's requirement.

Figure 17.16 compares the distribution of water resistance levels for individual jackets with the customer specifications. The distribution of water resistance is approximately Normal, and we estimate its mean to be very close to 2750 mm and the standard deviation to be about 384 mm. The distribution is safely within the specifications.

Times change, however. The outfitting company demands more similarity in jackets and decides to require that the water resistance level lie between 1500 and 3500 mm. These new specification limits also appear in Figure 17.16. The process is not capable of meeting the new requirements. The process remains in control. The change in its capability is entirely due to a change in external requirements.

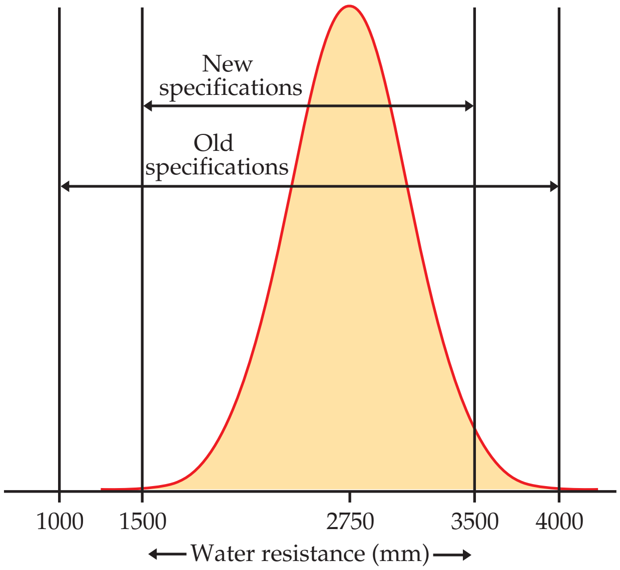


FIGURE 17.16 Comparison of the distribution of water resistance (Normal curve) with original and tightened specifications, Example 17.14. The process in its current state is not capable of meeting the new specifications.

Because the waterproofing process is in control, we know that it is not capable of meeting the new specifications. That's an advantage of control, but the fact remains that control does not guarantee capability. We will discuss numerical measures of capability in Section 17.3.

Managers must understand that *if a process that is in control does not have adequate capability, fundamental changes in the process are needed*. The process is doing as well as it can and displays only the chance variation that is natural to its present state. Slogans to encourage the workers or disciplining the workers for poor performance will not change the state of the process. Better training for workers is a change in the process that may improve capability. New equipment or more uniform material may also help, depending on the findings of a careful investigation.

SECTION 17.2 SUMMARY

- An ***R* chart** based on the **range** of observations in a sample is often used in place of an s chart. Interpret $\bar{x}$ and R charts exactly as you would interpret $\bar{x}$ and s charts.

- It is common to use **out-of-control rules** in addition to "one point outside the control limits." In particular, a **runs rule** for the $\bar{x}$ chart allows the chart to respond more quickly to a gradual drift in the process center.

- **Control charts based on past data** are used at the **chart setup** stage for a process that may not be in control. Start with control limits calculated from the same past data that you are plotting. Beginning with the s chart, narrow the limits as you find special causes, and remove the points influenced by these causes. When the remaining points are in control, use the resulting limits to monitor the process.

- Statistical process control maintains quality more economically than inspecting the final output of a process. Samples that are **rational subgroups** are important to effective control charts. A process in control is stable so that we can predict its behavior. If individual measurements have a Normal distribution, we can give the **natural tolerances**.

- A process is **capable** if it can meet the requirements placed on it. Control (stability over time) does not in itself imply capability. Remember that control describes the internal state of the process, whereas capability relates the state of the process to external specifications.

SECTION 17.2 EXERCISES

For Exercise 17.26, see page 17-24; for Exercises 17.27 and 17.28, see page 17-26; and for Exercises 17.29 and 17.30, see page 17-31.

17.31 Setting up a control chart. In Exercise 17.12 (page 17-18), the $\bar{x}$ and s control charts for the placement of the rum label were based on historical results. Suppose that a new labeling machine has been purchased and new control limits need to be determined. Table 17.7 contains the means and standard deviations of the first 24 batch samples. We will use these to determine tentative control limits. LABEL

(a) Estimate the center line and control limits for the s chart using all 24 samples.

(b) Does the variation within samples appear to be in control? If not, remove any out-of-control samples and recalculate the limits. We'll assume that any out-of-control samples are due to the operators adjusting to the new machine.

(c) Using the remaining samples, estimate the center line and control limits for the $\bar{x}$ chart. Again remove any out-of-control samples and recalculate.

(d) How do these control limits compare with the ones obtained in Exercise 17.12?

17.32 Setting up another control chart. Refer to the previous exercise. Table 17.8 contains another set

TABLE 17.7 $\bar{x}$ and s for 24 Samples of Label Placement (in inches)

Sample	$\bar{x}$	s	Sample	$\bar{x}$	s
1	1.9824	0.0472	13	1.9949	0.0964
2	2.0721	0.0479	14	2.0287	0.0607
3	2.0031	0.0628	15	1.9391	0.0481
4	2.0088	0.1460	16	1.9801	0.1133
5	2.0445	0.0850	17	1.9991	0.0482
6	2.0322	0.0676	18	1.9834	0.0572
7	2.0209	0.0651	19	2.0348	0.0734
8	1.9927	0.1291	20	1.9935	0.0584
9	2.0164	0.0889	21	1.9866	0.0628
10	2.0462	0.0662	22	1.9599	0.0829
11	2.0438	0.0554	23	2.0018	0.0541
12	2.0269	0.0493	24	1.9954	0.0566

TABLE 17.8 $\bar{x}$ and s for 24 Samples of Label Placement (in inches)

Sample	$\bar{x}$	s	Sample	$\bar{x}$	s
1	2.0309	0.1661	13	1.9907	0.0620
2	2.0066	0.1366	14	1.9612	0.0748
3	2.0163	0.0369	15	2.0312	0.0421
4	2.0970	0.1088	16	2.0293	0.0932
5	1.9499	0.0905	17	1.9758	0.0252
6	1.9859	0.1683	18	2.0255	0.0728
7	1.9456	0.0920	19	1.9574	0.0186
8	2.0213	0.0478	20	2.0320	0.0151
9	1.9621	0.0489	21	1.9775	0.0294
10	1.9529	0.0456	22	1.9612	0.0911
11	1.9995	0.0519	23	2.0042	0.0365
12	1.9927	0.0762	24	1.9933	0.0293

of 24 samples. Repeat parts (a), (b), and (c) using this data set. LABEL1

17.33 Control chart for an unusual sampling situation. Invoices are processed and paid by two clerks, one very experienced and the other newly hired. The experienced clerk processes invoices quickly. The new hire often refers to the procedures handbook and is much slower. Both are quite consistent so that their times vary little from invoice to invoice. Suppose that each daily sample of four invoice processing times comes from only one of the clerks. Thus, some samples are from one and some from the other clerk. Sketch the $\bar{x}$ chart pattern that will result.

17.34 Altering the sampling plan. Refer to Exercise 17.33. Suppose instead that each sample contains an equal number of invoices from each clerk.

(a) Sketch the $\bar{x}$ and s chart patterns that will result.

(b) The process in this case will appear in control. When might this be an acceptable conclusion?

17.35 Reevaluating the process parameters. The $\bar{x}$ and s control charts for the waterproofing example were based on $\mu = 2750$ mm and $\sigma = 430$ mm. Table 17.1 (page 17-10) gives the 20 most recent samples from this process. H2ORES)

(a) Estimate the process μ and σ based on these 20 samples.

(b) Your calculations suggest that the process σ may now be less than 430 mm. Explain why the s chart in Figure 17.7 (page 17-15) suggests the same conclusion. (If this pattern continues, we would eventually update the value of σ used for control limits.)

17.36 Estimating the control chart limits from past data. Table 17.9 gives data on the losses (in dollars) incurred by a hospital in treating DRG 209 (major joint replacement) patients.[11] The hospital has taken from its records a random sample of eight such patients each month for 15 months. DRG

(a) Make an s control chart using center lines and limits calculated from these past data. There are no points out of control.

(b) Because the s chart is in control, base the $\bar{x}$ chart on all 15 samples. Make this chart. Is it also in control?

17.37 Efficient process control. A company that makes cellular phones requires that its microchip supplier practice statistical process control and submit control charts for verification. This allows the company to eliminate inspection of the microchips as they arrive, a considerable cost savings. Explain carefully why incoming inspection can safely be eliminated.

17.38 Determining the tolerances for losses from DRG 209 patients. Table 17.9 gives data on hospital losses for samples of DRG 209 patients. The distribution of losses has been stable over time. What are the natural tolerances within which you expect losses on nearly all such patients to fall? DRG

17.39 Checking the Normality of losses. Do the losses on the 120 individual patients in Table 17.9 appear to come from a single Normal distribution? Make a Normal quantile plot and discuss what it shows. Are the natural tolerances you found in the previous exercise trustworthy? Explain your answer. DRG

17.40 The percent of products that meet specifications. If the water resistance readings of individual jackets follow

TABLE 17.9 Hospital Losses for 15 Samples of DRG 209 Patients

Sample	Loss (dollars)								Sample mean	Standard deviation
1	6835	5843	6019	6731	6362	5696	7193	6206	6360.6	521.7
2	6452	6764	7083	7352	5239	6911	7479	5549	6603.6	817.1
3	7205	6374	6198	6170	6482	4763	7125	6241	6319.8	749.1
4	6021	6347	7210	6384	6807	5711	7952	6023	6556.9	736.5
5	7000	6495	6893	6127	7417	7044	6159	6091	6653.2	503.7
6	7783	6224	5051	7288	6584	7521	6146	5129	6465.8	1034.3
7	8794	6279	6877	5807	6076	6392	7429	5220	6609.2	1104.0
8	4727	8117	6586	6225	6150	7386	5674	6740	6450.6	1033.0
9	5408	7452	6686	6428	6425	7380	5789	6264	6479.0	704.7
10	5598	7489	6186	5837	6769	5471	5658	6393	6175.1	690.5
11	6559	5855	4928	5897	7532	5663	4746	7879	6132.4	1128.6
12	6824	7320	5331	6204	6027	5987	6033	6177	6237.9	596.6
13	6503	8213	5417	6360	6711	6907	6625	7888	6828.0	879.8
14	5622	6321	6325	6634	5075	6209	4832	6386	5925.5	667.8
15	6269	6756	7653	6065	5835	7337	6615	8181	6838.9	819.5

a Normal distribution, we can describe capability by giving the percent of jackets that meet specifications. The old specifications for water resistance are 1000 to 4000 mm. The new specifications are 1500 to 3500 mm. Because the process is in control, we can estimate (Example 17.13) that water resistance has mean 2750 mm and standard deviation 384 mm. H2ORES

(a) What percent of jackets meet the old specifications?

(b) What percent meet the new specifications?

17.41 Improving the capability of the process. Refer to the previous exercise. The center of the specifications for waterproofing is 2500 mm, but the center of our process is 2750 mm. We can improve capability by adjusting the process to have center 2500 mm. This is an easy adjustment that does not change the process variation. What percent of jackets now meet the new specifications?

17.42 Monitoring the calibration of a densitometer. Loss of bone density is a serious health problem for many people, especially older women. Conventional X-rays often fail to detect loss of bone density until the loss reaches 25% or more. New equipment, such as the Lunar bone densitometer, is much more sensitive. A health clinic installs one of these machines. The manufacturer supplies a "phantom," an aluminum piece of known density that can be used to keep the machine calibrated. Each morning, the clinic makes two measurements on the phantom before measuring the first patient. Control charts based on these measurements alert the operators if the machine has lost calibration. Table 17.10 contains data for the first 30 days of operation.[12] The units are grams per square centimeter (for technical reasons, area rather than volume is measured). DENSITY

TABLE 17.10 Daily Calibration Samples for a Lunar Bone Densitometer

Day	Measurements (g/cm^2)		$\bar{x}$	s
1	1.261	1.260	1.2605	0.000707
2	1.261	1.268	1.2645	0.004950
3	1.258	1.261	1.2595	0.002121
4	1.261	1.262	1.2615	0.000707
5	1.259	1.262	1.2605	0.002121
6	1.269	1.260	1.2645	0.006364
7	1.262	1.263	1.2625	0.000707
8	1.264	1.268	1.2660	0.002828
9	1.258	1.260	1.2590	0.001414
10	1.264	1.265	1.2645	0.000707
11	1.264	1.259	1.2615	0.003536
12	1.260	1.266	1.2630	0.004243
13	1.267	1.266	1.2665	0.000707
14	1.264	1.260	1.2620	0.002828
15	1.266	1.259	1.2625	0.004950
16	1.257	1.266	1.2615	0.006364
17	1.257	1.266	1.2615	0.006364
18	1.260	1.265	1.2625	0.003536
19	1.262	1.266	1.2640	0.002828
20	1.265	1.266	1.2655	0.000707
21	1.264	1.257	1.2605	0.004950
22	1.260	1.257	1.2585	0.002121
23	1.255	1.260	1.2575	0.003536
24	1.257	1.259	1.2580	0.001414
25	1.265	1.260	1.2625	0.003536
26	1.261	1.264	1.2625	0.002121
27	1.261	1.264	1.2625	0.002121
28	1.260	1.262	1.2610	0.001414
29	1.260	1.256	1.2580	0.002828
30	1.260	1.262	1.2610	0.001414

(a) Calculate $\bar{x}$ and s for the first two days to verify the table entries for those quantities.

(b) What kind of variation does the s chart monitor in this setting? Make an s chart and comment on control. If any points are out of control, remove them and recompute the chart limits until all remaining points are in control. (That is, assume that special causes are found and removed.)

(c) Make an $\bar{x}$ chart using the samples that remain after you have completed part (b). What kind of variation will be visible on this chart? Comment on the stability of the machine over these 30 days based on both charts.

17.43 Determining the natural tolerances for the distance between holes. Figure 17.10 (page 17-22) displays a record sheet for 18 samples of distances between mounting holes in an electrical meter. In Exercise 17.21 (page 17-21), you found that Sample 5 was out of control on the process-monitoring s chart. The special cause responsible was found and removed. Based on the 17 samples that were in control, what are the natural tolerances for the distance between the holes? MOUNT

17.44 Determining the natural tolerances for the densitometer. Remove any samples in Table 17.10 that your work in Exercise 17.42 showed to be out of control on either chart. Estimate the mean and standard deviation of individual measurements on the phantom. What are the natural tolerances for these measurements? DENSITY

17.45 Determining the percent of meters that meet specifications. The record sheet in Figure 17.10 gives the specifications as 0.6054 ± 0.0010 inch. That's 54 ± 10 as the data are coded on the record. Assuming that the distance varies Normally from meter to meter, about what percent of meters meet the specifications? DENSITY

17.46 Assessing the Normality of the densitometer measurements. Are the 60 individual measurements in Table 17.10 at least approximately Normal so that the natural tolerances you calculated in Exercise 17.44 can be trusted? Make a Normal quantile plot (or another graph if your software is limited) and discuss what you see. DENSITY

17.47 Assessing the Normality of the distance between holes. Make a Normal quantile plot of the 85 distances in the data file MOUNT that remain after removing Sample 5. How does the plot reflect the limited precision of the measurements (all of which end in 4)? Is there any departure from Normality that would lead you to discard your conclusion from Exercise 17.43? (If your software will not make Normal quantile plots, use a histogram to assess Normality.) MOUNT

17.48 Determining the natural tolerances for the weight of ground beef. Table 17.3 (page 17-19) gives data on the weight of ground beef sections. Because the distribution of weights has been stable, use the data in Table 17.3 to construct the natural tolerances within which you expect almost all the weights to fall. MEATWGT

17.49 Assessing the Normality of the weight measurements. Refer to the previous exercise. Do the weights of the 60 individual sections in Table 17.3 appear to come from a single Normal distribution? Make a Normal quantile plot and discuss whether the natural tolerances you found in the previous exercise are trustworthy. MEATWGT

17.50 Control charts for the bore diameter of a bearing. A sample of five skateboard bearings is taken near the end of each hour of production. Table 17.11 gives $\bar{x}$ and s for the first 21 samples, coded in units of 0.001 mm from the target value. The specifications allow a range of ± 0.004 mm about the target (a range of -4 to $+4$ as coded). BEARINGS

(a) Make an s chart based on past data and comment on control of short-term process variation.

(b) Because the data are coded about the target, the process mean for the data provided is $\mu = 0$. Make an $\bar{x}$ chart and comment on control of long-term process variation. What special $\bar{x}$-type cause probably explains the lack of control of $\bar{x}$?

17.51 Detecting special cause variation. Is each of the following examples of a special cause most likely to first result in (i) a sudden change in level on the s or R chart, (ii) a sudden change in level on the $\bar{x}$ chart, or (iii) a gradual drift up or down on the $\bar{x}$ chart? In each case, briefly explain your reasoning.

(a) An airline pilots' union puts pressure on management during labor negotiations by asking its members to "work to rule" in doing the detailed checks required before a plane can leave the gate.

TABLE 17.11 $\bar{x}$ and s for Samples of Bore Diameter

Sample	$\bar{x}$	s	Sample	$\bar{x}$	s
1	0.0	1.225	12	0.8	3.899
2	0.4	1.517	13	2.0	1.581
3	0.6	2.191	14	0.2	2.049
4	1.0	3.162	15	0.6	2.302
5	−0.8	2.280	16	1.2	2.588
6	−1.0	2.345	17	2.8	1.924
7	1.6	1.517	18	2.6	3.130
8	1.0	1.414	19	1.8	2.387
9	0.4	2.608	20	0.2	2.775
10	1.4	2.608	21	1.6	1.949
11	0.8	1.924			

(b) Measurements of part dimensions that were formerly made by hand are now made by a very accurate laser system. (The process producing the parts does not change—measurement methods can also affect control charts.)

(c) Inadequate air conditioning on a hot day allows the temperature to rise during the afternoon in an office that prepares a company's invoices.

17.52 Deming speaks. The following comments were made by the quality guru W. Edwards Deming (1900–1993).[13] Choose one of these sayings. Explain carefully what facts about improving quality the saying attempts to summarize.

(a) "People work in the system. Management creates the system."

(b) "Putting out fires is not improvement. Finding a point out of control, finding the special cause and removing it, is only putting the process back to where it was in the first place. It is not improvement of the process."

(c) "Eliminate slogans, exhortations and targets for the workforce asking for zero defects and new levels of productivity."

17.53 Monitoring the winning times of the Boston Marathon. The Boston Marathon has been run each year since 1897. Winning times were highly variable in the early years, but control improved as the best runners became more professional. A clear downward trend continued until the 1980s. Sam plans to make a control chart for the winning times from 1980 to the present. Calculation from the winning times from 1980 to 2015 gives

$$\bar{x} = 129.49 \text{ minutes} \quad \text{and} \quad s = 2.13 \text{ minutes}$$

Sam draws a center line at $\bar{x}$ and control limits at $\bar{x} \pm 3s$ for a plot of individual winning times. Explain carefully why these control limits are too wide to effectively signal unusually fast or slow times.

17.54 Monitoring weight. Joe has recorded his weight, measured at the gym after a workout, for several years. The mean is 181 pounds and the standard deviation is 1.7 pounds, with no signs of lack of control. An injury keeps Joe away from the gym for several months. The data below give his weight, measured once each week for the first 16 weeks after he returns from the injury:

Week	1	2	3	4	5	6	7	8
Weight	185.2	185.5	186.3	184.3	183.1	180.8	183.8	182.1
Week	9	10	11	12	13	14	15	16
Weight	181.1	180.1	178.7	181.2	183.1	180.2	180.8	182.2

Joe wants to plot these individual measurements on a control chart. When each "sample" is just one measurement, short-term variation is estimated by advanced techniques.[14] The short-term variation in Joe's weight is estimated to be about $\sigma = 1.6$ pounds. Joe has a target of $\mu = 181$ pounds for his weight. Make a control chart for his measurements, using control limits $\mu \pm 2\sigma$. It is common to use these narrower limits on an "individuals chart." Comment on individual points out of control and on runs. Is Joe's weight stable or does it change systematically over this period? JOEWGT

17.3 Process Capability Indexes

When you complete this section, you will be able to:

- Estimate the percent of product that meets specifications using the Normal distribution.
- Explain why the percent of product meeting specifications is not a good measure of capability.
- Compute and interpret the C_p and C_{pk} capability indexes.
- Identify issues that affect the interpretation of capability indexes.

Capability describes the quality of the output of a process relative to the needs or requirements of the users of that output. To be more precise, capability relates the *actual performance* of a process in control, after special causes have been removed, to the *desired* performance.

Suppose, to take a simple but common setting, that there are *specifications* set for some characteristic of the process output. The viscosity of the elastomer in Example 17.11 (page 17-26) is supposed to be 45 ± 5 Mooneys. The speed with which calls are answered at a corporate customer service call center is supposed to be no more than 30 seconds.

In this setting, we might measure capability by the *percent of output that meets the specifications*. When the variable we measure has a Normal distribution, we can estimate this percent using the mean and standard deviation estimated from past control chart samples. When the variable is not Normal, we can use the actual percent of the measurements in the samples that meet the specifications.

EXAMPLE 17.15

What is the probability of meeting specifications? **(a)** Before concluding the process improvement study begun in Example 17.11, we found and fixed special causes and eliminated from our data the samples on which those causes operated. The remaining viscosity measurements have $\bar{\bar{x}} = 48.7$ and $s = 0.85$. Note once again that to draw conclusions about viscosity for individual lots we estimate the standard deviation σ from all individual lots, not from the average $\bar{s}$ of sample standard deviations.

The specifications call for the viscosity of the elastomer to lie in the range 45 ± 5. A Normal quantile plot shows the viscosities to be quite Normal. Figure 17.17(a) shows the Normal distribution of lot viscosities with the specification limits 45 ± 5. These are marked **LSL** for *lower specification limit* and **USL** for *upper specification limit*. The percent of lots that meet the specifications is about

LSL

USL

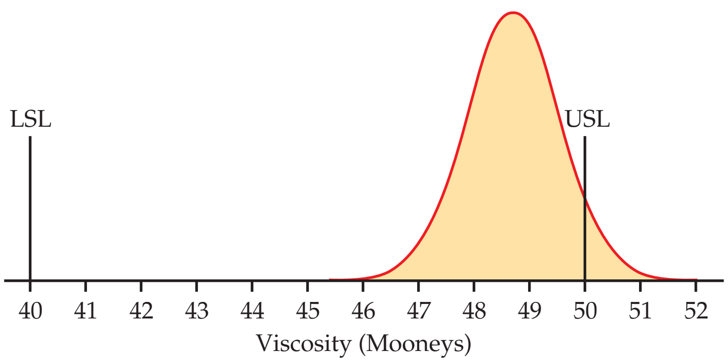
LOOK BACK

Normal distribution calculations, p. 61

$$P(40 \le \text{viscosity} \le 50) = P\left(\frac{40 - 48.7}{0.85} \le Z \le \frac{50 - 48.7}{0.85}\right)$$
$$= P(-10.2 \le Z \le 1.53) = 0.937$$

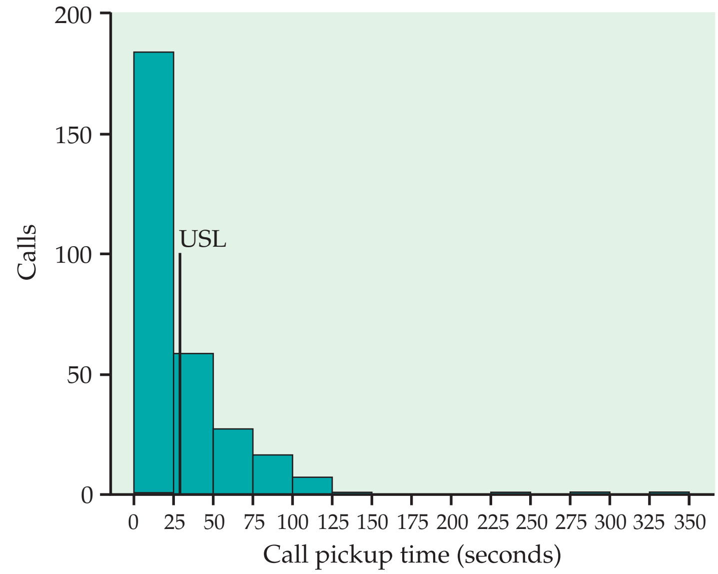
Roughly 94% of the lots meet the specifications. If we can adjust the process center to the center of the specifications, $\mu = 45$, it is clear from Figure 17.17(a) that essentially 100% of lots will meet the specifications.

(b) Times to answer calls to a corporate customer service center are usually right-skewed. Figure 17.17(b) is a histogram of the times for 300 calls to the call center of a large bank.[15] The specification limit of 30 seconds is marked USL. The median is 20 seconds, but the mean is 32 seconds. Of the 300 calls, 203 were answered in no more than 30 seconds. That is, 203/300 = 68% of the times meet the specifications.



(a)

FIGURE 17.17 Comparing distributions of individual measurements with specifications, Example 17.15. (a) Viscosity has a Normal distribution. The capability is poor but will be good if we can properly center the process. (b) Response times to customer calls have a right-skewed distribution and only an upper specification limit. Capability is again poor.



(b)

FIGURE 17.17 *Continued*

Turns out, however, that the percent meeting specifications is a poor measure of capability. Figure 17.18 shows why. This figure compares the distributions of the diameter of the same part manufactured by two processes. The target diameter and the specification limits are marked. All the parts produced by Process A meet the specifications, but about 1.5% of those from Process B fail to do so.

Nonetheless, Process B appears superior to Process A because it is less variable: much more of Process B's output is close to the target. Process A produces many parts close to LSL and USL. These parts meet the specifications, but they will likely fit and perform more poorly than parts with diameters close to the center of the specifications. A distribution like that for Process A might

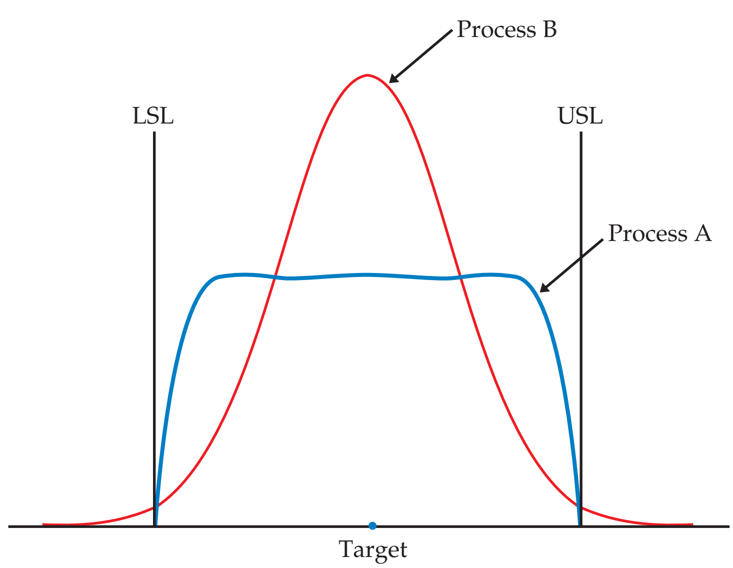


FIGURE 17.18 Two distributions for part diameters. All the parts from Process A meet the specifications, but a higher proportion of parts from Process B have diameters close to the target.

result from inspecting all the parts and discarding those whose diameters fall outside the specifications. That's not an efficient way to achieve quality.

We need a way to measure process capability that pays attention to the variability of the process (smaller is better). The standard deviation does that, but it doesn't measure capability because it takes no account of the specifications that the output must meet.

Capability indexes start with the idea of comparing process variation with the specifications. Process B will beat Process A by such a measure. Capability indexes also allow us to measure process improvement—we can continue to drive down variation, and so improve the process, long after 100% of the output meets specifications. Continual improvement of processes is our goal, not just reaching "satisfactory" performance. The real importance of capability indexes is that they give us numerical measures to describe ever-better process quality.

The capability indexes C_p and C_{pk}

Capability indexes are numerical measures of process capability that, unlike percent meeting specifications, have no upper limit such as 100%. We can use capability indexes to measure continuing improvement of a process. Of course, reporting just one number has limitations. What is more, the usual indexes are based on thinking about Normal distributions. They are not meaningful for distinctly non-Normal output distributions like the call center response times in Figure 17.17(b).

CAPABILITY INDEXES

Consider a process with specification limits LSL and USL for some measured characteristic of its output. The process mean for this characteristic is μ and the standard deviation is σ. The **capability index C_p** is

$$C_p = \frac{\text{USL} - \text{LSL}}{6\sigma}$$

The **capability index C_{pk}** is

$$C_{pk} = \frac{|\mu - \text{nearer spec limit}|}{3\sigma}$$

Set $C_{pk} = 0$ if the process mean μ lies outside the specification limits. Large values of C_p or C_{pk} indicate more capable processes.

Capability indexes start from the fact that *Normal distributions are in practice about 6 standard deviations wide*. That's the 99.7 part of the 68–95–99.7 rule. Conceptually, C_p is the specification width as a multiple of the process width 6σ. When $C_p = 1$, the process output will just fit within the specifications if the center is midway between LSL and USL.

Larger values of C_p are better—the process output can fit within the specs with room to spare. But a process with high C_p can produce poor-quality product if it is not correctly centered.

C_{pk} remedies this deficiency by considering both the center μ and the variability σ of the measurements. The denominator 3σ in C_{pk} is half the process width. It is the space needed on either side of the mean if essentially all the output is to lie between LSL and USL. When $C_{pk} = 1$, the process has just this much space between the mean and the nearer of LSL and USL. Again, higher values are better. C_{pk} is the most common capability index, but starting with C_p helps us see how the indexes work.

EXAMPLE 17.16

A comparison of the C_p and C_{pk} indexes. Consider the series of pictures in Figure 17.19. We might think of a process that machines a metal part. Measure a dimension of the part that has LSL and USL as its specification limits. As usual, there is variation from part to part. The dimensions vary Normally with mean μ and standard deviation σ.

Figure 17.19(a) shows process width equal to the specification width. That is, $C_p = 1$. Almost all the parts will meet specifications *if,* as in this figure, the process mean μ is at the center of the specs. Because the mean is centered, it is 3σ from both LSL and USL, so $C_{pk} = 1$ also. In Figure 17.19(b), the mean has moved down to LSL. Only half the parts will meet the specifications. C_p is unchanged because the process width has not changed. But C_{pk} sees that the center μ is right on the edge of the specifications, $C_{pk} = 0$. The value remains 0 if μ moves outside the specifications.

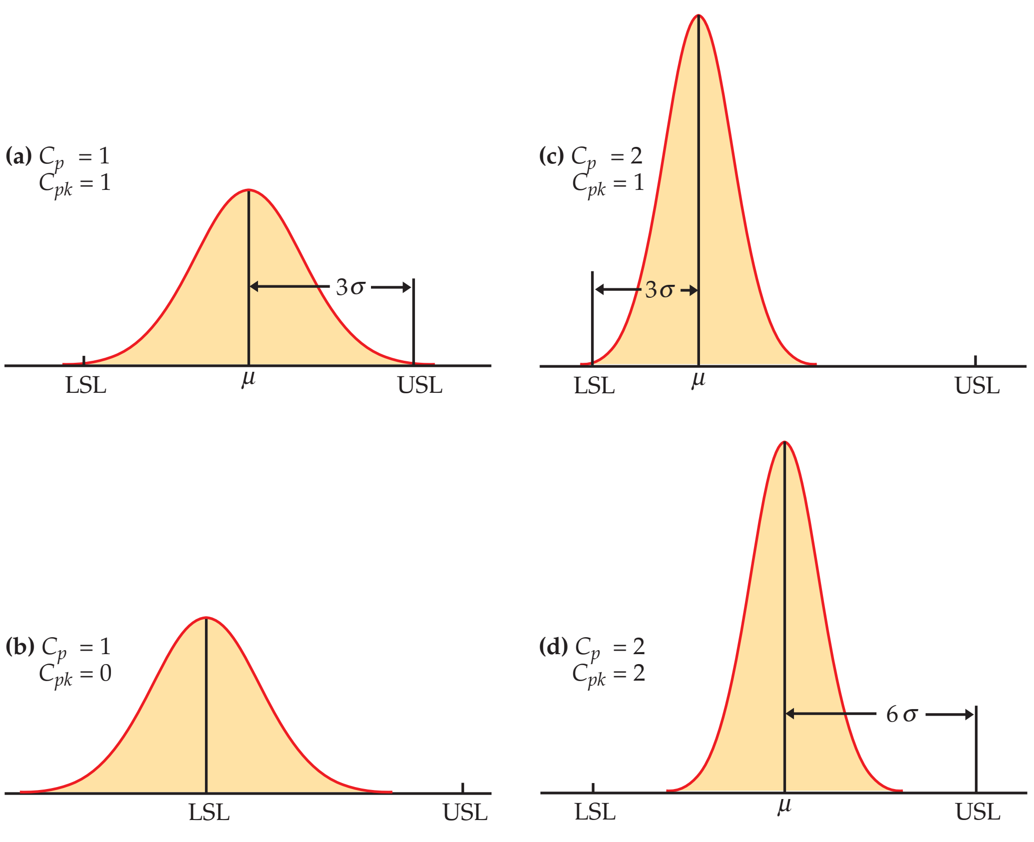


FIGURE 17.19 How capability indexes work. (a) Process centered, process width equal to specification width. (b) Process off-center, process width equal to specification width. (c) Process off-center, process width equal to half the specification width. (d) Process centered, process width equal to half the specification width.

In Figures 17.19(c) and (d), the process σ has been reduced to half the value it had in panels (a) and (b). The process width 6σ is now half the specification width, so $C_p = 2$. In Figure 17.19(c), the center is just 3 of the new σ's above LSL, so that $C_{pk} = 1$. Figure 17.19(d) shows the same smaller σ accompanied by mean μ correctly centered between LSL and USL. C_{pk} rewards the process for moving the center from 3σ to 6σ away from the nearer limit by increasing from 1 to 2. You see that C_p and C_{pk} are equal if the process is properly centered. If not, C_{pk} is smaller than C_p.

EXAMPLE 17.17

Computing C_p and C_{pk} for the viscosity process. Figure 17.17(a) compares the distribution of the viscosities of lots of elastomers with the specifications LSL = 40 and USL = 50. The distribution here, as is always true in practice, is *estimated* from past observations on the process. The estimates are

$$\hat{\mu} = \bar{\bar{x}} = 48.7$$
$$\hat{\sigma} = s = 0.85$$

Because capability describes the distribution of individual measurements, we once more estimate σ from individual measurements rather than using the estimate $\bar{s}/c_4$ that we employ for control charts.

These estimates may be quite accurate if we have data on many past lots. Estimates based on only a few observations may, however, be inaccurate because statistics from small samples can have large sampling variability. This important point is often not appreciated when capability indexes are used in practice. To emphasize that we can only estimate the indexes, we write $\hat{C}_p$ and $\hat{C}_{pk}$ for values calculated from sample data. They are

$$\hat{C}_p = \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}}$$
$$= \frac{50 - 40}{(6)(0.85)} = \frac{10}{5.1} = 1.96$$
$$\hat{C}_{pk} = \frac{|\hat{\mu} - \text{nearer limit}|}{3\hat{\sigma}}$$
$$= \frac{50 - 48.7}{(3)(0.85)} = \frac{1.3}{2.55} = 0.51$$

$\hat{C}_p = 1.96$ is quite satisfactory because it indicates that the process width is only about half the specification width. The small value of $\hat{C}_{pk}$ reflects the fact that the process center is not close to the center of the specs. If we can move the center μ to 45, then $\hat{C}_{pk}$ will also be 1.96.

USE YOUR KNOWLEDGE

17.55 Specification limits versus control limits. The manager you report to is confused by LSL and USL versus LCL and UCL. The notations look similar. Carefully explain the conceptual difference between specification limits for individual measurements and control limits for $\bar{x}$.

17.56 Interpreting the capability indexes. Sketch Normal curves that represent measurements on products from a process with

(a) $C_p = 1.0$ and $C_{pk} = 0.5$.

(b) $C_p = 1.0$ and $C_{pk} = 1.0$.

(c) $C_p = 2.0$ and $C_{pk} = 1.0$.

Cautions about capability indexes

Capability indexes are widely used, especially in manufacturing. Some large manufacturers even set standards, such as $C_{pk} \geq 1.33$, that their suppliers must meet. That is, suppliers must show that their processes are in control (through control charts) and also that they are capable of high quality (as measured by C_{pk}). There are good reasons for requiring C_{pk}: it is a better description of process quality than "100% of output meets specs," and it can document continual improvement. Nonetheless, it is easy to trust C_{pk} too much. We will point to three possible pitfalls.

How to cheat on C_{pk} Estimating C_{pk} requires estimates of the process mean μ and standard deviation σ. The estimates are usually based on samples measured in order to keep control charts. There is only one reasonable estimate of μ. This is the mean $\bar{x}$ of all measurements in recent samples, which is the same as the mean $\bar{\bar{x}}$ of the sample means.

There are two different ways of estimating σ, however. The standard deviation s of all measurements in recent samples will usually be larger than the control chart estimate $\bar{s}/c_4$ based on averaging the sample standard deviations. For C_{pk}, the proper estimate is s because we want to describe all the variation in the process output. Larger C_{pk}'s are better, and a supplier wanting to satisfy a customer can make C_{pk} a bit larger simply by using the smaller estimate $\bar{s}/c_4$ for σ. That's cheating.

Non-Normal distributions Many business processes, and some manufacturing processes as well, give measurements that are clearly right-skewed rather than approximately Normal. Measuring the times required to deal with customer calls or prepare invoices typically gives a right-skewed distribution—there are many routine cases and a few unusual or difficult situations that take much more time. Other processes have "heavy tails," with more measurements far from the mean than in a Normal distribution.

Process capability concerns the behavior of individual outputs, so the central limit theorem effect that improves the Normality of $\bar{x}$ does not help us. Capability indexes are, therefore, more strongly affected by non-Normality than are control charts. *It is hard to interpret C_{pk} when the measurements are strongly non-Normal.* Until you gain experience, it is best to apply capability indexes only when Normal quantile plots show that the distribution is at least roughly Normal.

Sampling variation All statistics are subject to sampling variation. If we draw another sample from the same process at the same time, we get slightly different $\bar{x}$ and s due to the luck of the draw in choosing samples. In process control language, the samples differ due to the common cause variation that is always present.

C_p and C_{pk} are, in practice, calculated from process data because we don't know the true process mean and standard deviation. That is, these capability indexes are statistics subject to sampling variation. A supplier under pressure from a large customer to measure C_{pk} often may base calculations on small samples from the process. The resulting estimate $\hat{C}_{pk}$ can differ from the true process C_{pk} in either direction.

EXAMPLE 17.18

Can we adequately measure C_{pk}? Suppose that the process of waterproofing is in control at its original level. Water resistance measurements are Normally distributed with mean $\mu = 2750$ mm and standard deviation $\sigma = 430$ mm. The tightened specification limits are LSL = 1500 and USL = 3500, so the true capability is

$$C_{pk} = \frac{3500 - 2750}{(3)(430)} = 0.58$$

Suppose also that the manufacturer measures four jackets each four-hour shift and then calculates $\hat{C}_{pk}$ at the end of eight shifts. That is, $\hat{C}_{pk}$ uses measurements from 32 jackets.

Figure 17.20 is a histogram of 24 computer-simulated $\hat{C}_{pk}$'s from this setting. They vary from 0.44 to 0.84, almost a two-to-one spread. It is clear that 32 measurements are not enough to reliably estimate C_{pk}.

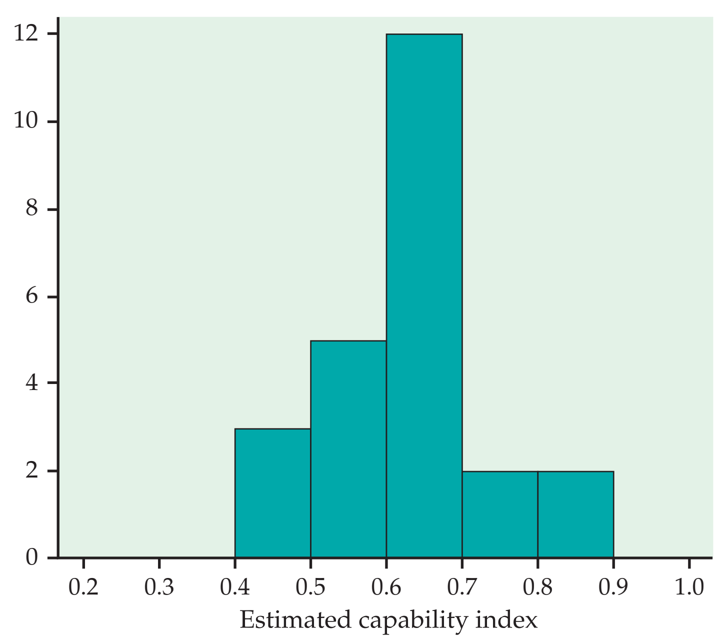


FIGURE 17.20 Capability indexes estimated from samples will vary from sample to sample. The histogram shows the variation in $\hat{C}_{pk}$ in 24 samples, each of size 32, Example 17.18. The process capability is in fact $C_{pk} = 0.58$.

CAUTION

As a very rough rule of thumb, don't trust $\hat{C}_{pk}$ unless it is based on at least 100 measurements.

SECTION 17.3 SUMMARY

- **Capability indexes** measure process variability (**C_p**) or process center and variability (**C_{pk}**) against the standard provided by external specifications for the output of the process. Larger values indicate higher capability.
- Interpretation of C_p and C_{pk} requires that measurements on the process output have a roughly Normal distribution. These indexes are not meaningful unless the process is in control so that its center and variability are stable.
- Estimates of C_p and C_{pk} can be quite inaccurate when based on small numbers of observations, due to sampling variability. You should mistrust estimates not based on at least 100 measurements.

SECTION 17.3 EXERCISES

For Exercises 17.55 and 17.56, see pages 17-45–17-46.

17.57 Capability indexes for the waterproofing process. Table 17.1 (page 17-10) gives 20 process control samples of the water resistance of a particular outdoor jacket. In Example 17.13, we estimated from these samples that $\hat{\mu} = \bar{\bar{x}} = 2750.7$ mm and $\hat{\sigma} = s = 383.8$ mm.

(a) The original specifications for water resistance were LSL = 1000 mm and USL = 4000 mm. Estimate C_p and C_{pk} for this process.

(b) A major customer tightened the specifications to LSL = 1500 mm and USL = 3500 mm. Now what are $\hat{C}_p$ and $\hat{C}_{pk}$?

17.58 Capability indexes for the waterproofing process, continued. We could improve the performance of the waterproofing process discussed in the previous exercise by making an adjustment that moves the center of the process to $\mu = 2500$ mm, the center of the specifications. We should do this, even if the original specifications remain in force, because this will require less sealer and, therefore, cost less. Suppose that we succeed in moving μ to 2500 with no change in the process variability σ, estimated by $s = 383.8$.

(a) What are $\hat{C}_p$ and $\hat{C}_{pk}$ with the original specifications? Compare the values with those from part (a) of the previous exercise.

(b) What are $\hat{C}_p$ and $\hat{C}_{pk}$ with the tightened specifications? Again compare with the previous results.

17.59 Capability indexes for the meat-packaging process. Table 17.3 (page 17-19) gives 20 process control samples of the weight of ground beef sections. The lower and upper specifications for the one-pound sections are 0.95 and 1.09. MEATWGT

(a) Using these data, estimate C_p and C_{pk} for this process.

(b) What may be a reason for the specifications being centered at a weight that is slightly greater than the desired one pound?

17.60 Can we improve the capability of the meat-packaging process? Refer to Exercise 17.59. The average weight of each section can be increased (or decreased) by increasing (or decreasing) the time between slices of the machine. Based on the results of the previous exercise, would a change in the slicing-time interval improve capability? If so, what value of the average weight should the company seek to attain, and what are $\hat{C}_p$ and $\hat{C}_{pk}$ with this new process mean?

17.61 Capability of a characteristic with a uniform distribution. Suppose that a quality characteristic has the uniform distribution on 0 to 1. Figure 17.21 shows the density curve. You can see that the process mean (the balance point of the density curve) is $\mu = 1/2$. The standard deviation turns out to be $\sigma = 0.289$. Suppose also that LSL = 1/4 and USL = 3/4.

(a) Mark LSL and USL on a sketch of the density curve. What is C_{pk}? What percent of the output meets the specifications?

(b) For comparison, consider a process with Normally distributed output having mean $\mu = 1/2$ and standard deviation $\sigma = 0.289$. This process has the same C_{pk} that you found in part (a). What percent of its output meets the specifications?

(c) What general fact do your calculations illustrate?

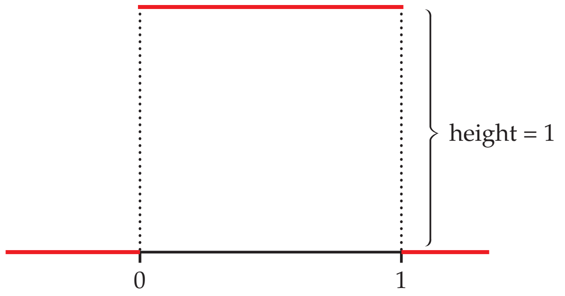


FIGURE 17.21 Density curve for the uniform distribution on 0 to 1, Exercise 17.61.

17.62 An alternative estimate for C_{pk} of the waterproofing process. In Exercise 17.58(b), you found $\hat{C}_{pk}$ for specifications LSL = 1500 and USL = 3500 using the standard deviation $s = 383.8$ for all 80 individual jackets in Table 17.1. Repeat the calculation using the control chart estimate $\hat{\sigma} = \bar{s}/c_4$. You should find this $\hat{C}_{pk}$ to be slightly larger.

17.63 Estimating capability indexes for the distance between holes. Figure 17.10 (page 17-22) displays a record sheet on which operators have recorded 18 samples of measurements on the distance between two mounting holes on an electrical meter. Sample 5 was out of control on an s chart. We remove it from the data after the special cause has been fixed. In Exercise 17.47 (page 17-39), you saw that the measurements are reasonably Normal. MOUNTING

(a) Based on the remaining 17 samples, estimate the mean and standard deviation of the distance between holes for the population of all meters produced by this process. Make a sketch comparing the Normal distribution with this mean and standard deviation with the specification limits 54 ± 10.

(b) What are $\hat{C}_p$ and $\hat{C}_{pk}$ based on the data? How would you characterize the capability of the process? (Mention both center and variability.)

17.64 Calculating capability indexes for the DRG 209 hospital losses. Table 17.9 (page 17-37) gives data on a hospital's losses for 120 DRG 209 patients, collected as 15 monthly samples of eight patients each. The process has been in control and losses have a roughly Normal distribution. The hospital decides that suitable specification limits for its loss in treating one such patient are LSL = \$4500 and USL = \$7500. DRG

(a) Estimate the percent of losses that meet the specifications.

(b) Estimate C_p.

(c) Estimate C_{pk}.

17.65 Assessing the capability of the skateboard bearings process. Recall the skateboard bearings process described in Exercise 17.50 (page 17-39). The bore diameter has specifications (7.9920, 8.000) mm. The process is monitored by $\bar{x}$ and s charts based on samples of five consecutive bearings each hour. Control has recently been excellent. The 200 individual measurements from the past week's 40 samples have

$$\bar{x} = 7.997 \text{ mm} \qquad s = 0.0025 \text{ mm}$$

A Normal quantile plot shows no important deviations from Normality.

(a) What percent of bearings will meet specifications if the process remains in its current state?

(b) Estimate the capability index C_{pk}.

17.66 Will these actions help the capability? Based on the results of the previous exercise, you conclude that the capability of the bearing-making process is inadequate. Here are some suggestions for improving the capability of this process. Comment on the usefulness of each action suggested.

(a) Narrowing the control limits so that the process is adjusted more often.

(b) Additional training of operators to ensure correct operating procedures.

(c) A capital investment program to install new fabricating machinery.

(d) An award program for operators who produce the fewest nonconforming bearings.

(e) Purchasing more uniform (and more expensive) metal stock from which to form the bearings.

17.67 C_p and "six sigma." A process with $C_p \geq 2$ is sometimes said to have "six-sigma quality." Sketch the specification limits and a Normal distribution of individual measurements for such a process when it is properly centered. Explain from your sketch why this is called six-sigma quality.

17.68 More on "six-sigma quality." The originators of the "six-sigma quality" idea reasoned as follows. Short-term process variation is described by σ. In the long term, the process mean μ will also vary. Studies show that, in most manufacturing processes, $\pm 1.5\sigma$ is adequate to allow for changes in μ. The six-sigma standard is intended to allow the mean μ to be as much as 1.5σ away from the center of the specifications and still meet high standards for percent of output lying outside the specifications.

(a) Sketch the specification limits and a Normal distribution for process output when $C_p = 2$ and the mean is 1.5σ away from the center of the specifications.

(b) What is C_{pk} in this case? Is six-sigma quality as strong a requirement as $C_{pk} \geq 2$?

(c) Because most people don't understand standard deviations, six-sigma quality is usually described as guaranteeing a certain level of parts per million of output that fails to meet specifications. Based on your sketch in part (a), what is the probability of an outcome outside the specification limits when the mean is 1.5σ away from the center? How many parts per million is this? (You will need software or a calculator for Normal probability calculations because the value you want is beyond the limits of the standard Normal table.)

Table 17.12 gives the process control samples that lie behind the histogram of call center response times in Figure 17.17(b) on page 17-42. A sample of six calls is

TABLE 17.12 Fifty Control Chart Samples of Call Center Response Times

Sample	Time (seconds)						Sample mean	Standard deviation
1	59	13	2	24	11	18	21.2	19.93
2	38	12	46	17	77	12	33.7	25.56
3	46	44	4	74	41	22	38.5	23.73
4	25	7	10	46	78	14	30.0	27.46
5	6	9	122	8	16	15	29.3	45.57
6	17	17	9	15	24	70	25.3	22.40
7	9	9	10	32	9	68	22.8	23.93
8	8	10	41	13	17	50	23.2	17.79
9	12	82	97	33	76	56	59.3	32.11
10	42	19	14	21	12	44	25.3	14.08
11	63	5	21	11	47	8	25.8	23.77
12	12	4	111	37	12	24	33.3	39.76
13	43	37	27	65	32	3	34.5	20.32
14	9	26	5	10	30	27	17.8	10.98
15	21	14	19	44	49	10	26.2	16.29
16	24	11	10	22	43	70	30.0	22.93
17	27	10	32	96	11	29	34.2	31.71
18	7	28	22	17	9	24	17.8	8.42
19	15	14	34	5	38	29	22.5	13.03
20	16	65	6	5	58	17	27.8	26.63
21	7	44	14	16	4	46	21.8	18.49
22	32	52	75	11	11	17	33.0	25.88
23	31	8	36	25	14	85	33.2	27.45
24	4	46	23	58	5	54	31.7	24.29
25	28	6	46	4	28	11	20.5	16.34
26	111	6	3	83	27	6	39.3	46.34
27	83	27	2	56	26	21	35.8	28.88
28	276	14	30	8	7	12	57.8	107.20
29	4	29	21	23	4	14	15.8	10.34
30	23	22	19	66	51	60	40.2	21.22
31	14	111	20	7	7	87	41.0	45.82
32	22	11	53	20	14	41	26.8	16.56
33	30	7	10	11	9	9	12.7	8.59
34	101	55	18	20	77	14	47.5	36.16
35	13	11	22	15	2	14	12.8	6.49
36	20	83	25	10	34	23	32.5	25.93
37	21	5	14	22	10	68	23.3	22.82
38	8	70	56	8	26	7	29.2	27.51
39	15	7	9	144	11	109	49.2	60.97
40	20	4	16	20	124	16	33.3	44.80
41	16	47	97	27	61	35	47.2	28.99
42	18	22	244	19	10	6	53.2	93.68
43	43	20	77	22	7	33	33.7	24.49
44	67	20	4	28	5	7	21.8	24.09
45	118	18	1	35	78	35	47.5	43.00
46	71	85	24	333	50	11	95.7	119.53
47	12	11	13	19	16	91	27.0	31.49
48	4	63	14	22	43	25	28.5	21.29
49	18	55	13	11	6	13	19.3	17.90
50	4	3	17	11	6	17	9.7	6.31

recorded each shift for quality improvement purposes. The time from the first ring until a representative answers the call is recorded. Table 17.12 gives data for 50 shifts, 300 calls total. Exercises 17.69, 17.70, and 17.71 make use of this setting. CALLS50

17.69 Choosing the sample. The six calls each shift are chosen at random from all calls received during the shift. Discuss the reasons behind this choice and those behind a choice to time six consecutive calls.

17.70 Constructing and interpreting the s chart. Table 17.12 also gives $\bar{x}$ and s for each of the 50 samples.

(a) Make an s chart and check for four points out of control.

(b) If the s-type cause responsible is found and removed, what would be the new control limits for the s chart? Verify that no points s are now out of control.

(c) Use the remaining 46 samples to find the center line and control limits for an $\bar{x}$ chart. Comment on the control (or lack of control) of $\bar{x}$. (Because the distribution of response times is strongly skewed, $\bar{s}$ is large and the control limits for $\bar{x}$ are wide. Control charts based on Normal distributions often work poorly when measurements are strongly skewed.)

17.71 More on interpreting the s chart. Each of the four out-of-control values of s in part (a) of the previous exercise is explained by a single outlier, a very long response time to one call in the sample. You can see these outliers in Figure 17.17(b). What are the values of these outliers, and what are the s-values for the four samples when the outliers are omitted? (The interpretation of the data is, unfortunately, now clear. Few customers will wait five minutes for a call to be answered, as the customer whose call took 333 seconds to answer did. We suspect that other customers hung up before their calls were answered. If so, response time data for the calls that were answered don't adequately picture the quality of service. We should now look at data on calls lost before being answered to see a fuller picture.)

17.4 Control Charts for Sample Proportions

When you complete this section, you will be able to:

- Distinguish when to use a p chart rather than an $\bar{x}$ chart.
- Compute the center line and control limits for a p chart and utilize the chart for process monitoring.

We have considered control charts for just one kind of data: measurements of a quantitative variable in some meaningful scale of units. We describe the distribution of measurements by its center and spread and use $\bar{x}$ and s or $\bar{x}$ and R charts for process control. There are control charts for other statistics that are appropriate for other kinds of data. The most common of these is the p chart for use when the data are proportions.

p CHART

A ***p* chart** is a control chart based on plotting sample proportions $\hat{p}$ from regular samples from a process against the order in which the samples were taken.

EXAMPLE 17.19

Examples of the *p* chart. Here are two examples of the usefulness of p charts:

Manufacturing. Measure two dimensions of a part and also grade its surface finish by eye. The part conforms if both dimensions lie within their specifications and the finish is judged acceptable. Otherwise, it is nonconforming. Plot the proportion of nonconforming parts in samples of parts from each shift.

School absenteeism. An urban school system records the percent of its eighth-grade students who are absent three or more days each month. Because students with high absenteeism in eighth grade often fail to complete high school, the school system has launched programs to reduce absenteeism. These programs include calls to parents of absent students, public-service messages to change community expectations, and measures to ensure that the schools are safe and attractive. A p chart will show if the programs are having an effect.

The manufacturing example illustrates an advantage of p charts: they can combine several specifications in a single chart. Nonetheless, *p charts have been rendered outdated in many manufacturing applications by improvements in typical levels of quality.* When the proportion of nonconforming parts is very small, even large samples of parts will rarely contain any bad parts. The sample proportions will almost all be 0, so that plotting them is uninformative.

It is better to choose important measured characteristics—voltage at a critical circuit point, for example—and keep $\bar{x}$ and s charts. Even if the voltage is satisfactory, quality can be improved by moving it yet closer to the exact voltage specified in the design of the part.

The school absenteeism example is a management application of p charts. More than 19% of all American eighth-graders miss three or more days of school per month, and this proportion is higher in large cities and for certain ethnic groups.[16] A p chart will be useful. Proportions of "things going wrong" are often higher in business processes than in manufacturing, so that p charts are an important tool in business.

Control limits for p charts

We studied the sampling distribution of a sample proportion $\hat{p}$ in Chapter 5. The center line and control limits for a 3σ control chart follow directly from the facts stated there, in the box on page 17-13. We ought to call such charts "$\hat{p}$ charts" because they plot sample proportions. Unfortunately, they have always been called p charts in quality control circles. We will keep the traditional name but also keep our usual notation: p is a *process* proportion and $\hat{p}$ is a *sample* proportion.

p CHART USING PAST DATA

Take regular samples from a process that has been in control. The samples need not all have the same size. Estimate the process proportion p of "successes" by

$$\bar{p} = \frac{\text{total number of successes in past samples}}{\text{total number of opportunities in these samples}}$$

The center line and control limits for a ***p* chart** for future samples of size n are

$$\text{UCL} = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$\text{CL} = \bar{p}$$

$$\text{LCL} = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

Common **out-of-control signals** are one sample proportion $\hat{p}$ outside the control limits or a run of nine sample proportions on the same side of the center line.

If we have k past samples of the *same* size n, then $\bar{p}$ is just the average of the k sample proportions. In some settings, you may meet samples of unequal size—differing numbers of students enrolled in a month or differing numbers of parts inspected in a shift. The average $\bar{p}$ estimates the process proportion p even when the sample sizes vary. Note that the control limits use the actual size n of a sample.

EXAMPLE 17.20

Monitoring employees' absences. Unscheduled absences by clerical and production workers are an important cost in many companies. Reducing the rate of absenteeism is, therefore, an important goal for a company's human relations department. A rate of absenteeism above 5% is a serious concern. Many companies set 3% absent as a desirable target. You have been asked to improve absenteeism in a production facility where 12% of the workers are now absent on a typical day.

You first do some background study—in greater depth than this very brief summary. Companies try to avoid hiring workers who are likely to miss work often, such as substance abusers. They may have policies that reward good attendance or penalize frequent absences by individual workers. Changing those policies in this facility will have to wait until the union contract is renegotiated. What might you do with the current workers under current policies?

Studies of absenteeism by clerical and production workers who do repetitive, routine work under close supervision point to unpleasant work environment and harsh or unfair treatment by supervisors as factors that increase absenteeism. It's now up to you to apply this general knowledge to your specific problem.

First, collect data. Daily absenteeism data are already available. You carry out a sample survey that asks workers about their absences and the reasons for them (responses are anonymous, of course). Workers who are more often absent complain about their supervisors and about the lighting at their workstations. Female workers complain that the rest rooms are dirty and unpleasant. You do more data analysis:

- A Pareto chart of average absenteeism rate for the past month broken down by supervisor (Figure 17.22) shows important differences among supervisors. Only supervisors B, E, and H meet the level of 5% or less absenteeism. Workers supervised by I and D have particularly high rates.
- Another Pareto chart (not shown) by type of workstation shows that a few types of workstation have high absenteeism rates.

Now you take action. You retrain all the supervisors in human relations skills, using B, E, and H as discussion leaders. In addition, a trainer works individually with supervisors I and D. You ask supervisors to talk with any absent worker when he or she returns to work. Working with the engineering department, you study the workstations with high absenteeism rates and make changes such as better lighting. You refurbish the rest rooms (for both sexes, even though only women complained) and schedule more frequent cleaning.

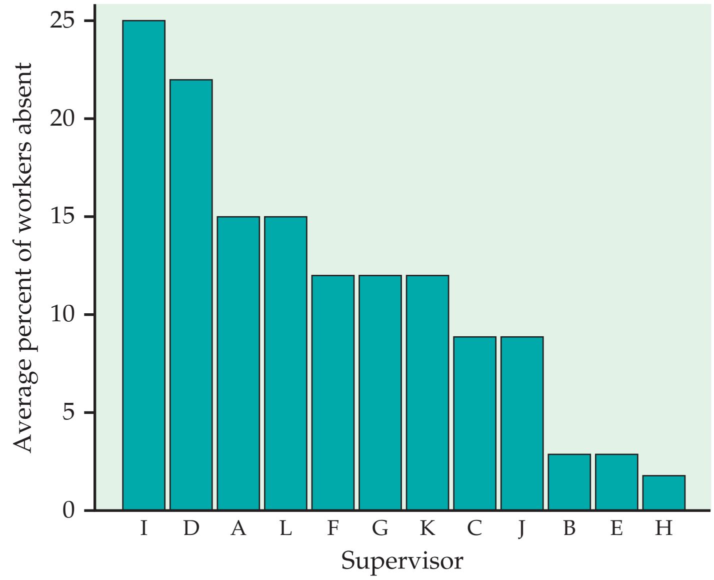


FIGURE 17.22 Pareto chart of the average absenteeism rate for workers reporting to each of 12 supervisors.

EXAMPLE 17.21

Are your actions effective? You hope to see a reduction in absenteeism. To view progress (or lack of progress), you will keep a p chart of the proportion of absentees. The plant has 987 production workers. For simplicity, you just record the number who are absent from work each day. Only unscheduled absences count, not planned time off such as vacations. Each day you will plot

$$\hat{p} = \frac{\text{number of workers absent}}{987}$$

You first look back at data for the past three months. There were 64 workdays in these months. The total workdays available for the workers was

$$(64)(987) = 63{,}168 \text{ person-days}$$

Absences among all workers totaled 7580 person-days. The average daily proportion absent was, therefore,

$$\bar{p} = \frac{\text{total days absent}}{\text{total days available for work}}$$
$$= \frac{7580}{63{,}168} = 0.120$$

The daily rate has been in control at this level.

These past data allow you to set up a p chart to monitor future proportions absent:

$$\text{UCL} = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.120 + 3\sqrt{\frac{(0.120)(0.880)}{987}}$$
$$= 0.120 + 0.031 = 0.151$$
$$\text{CL} = \bar{p} = 0.120$$
$$\text{LCL} = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.120 - 3\sqrt{\frac{(0.120)(0.880)}{987}}$$
$$= 0.120 - 0.031 = 0.089$$

Table 17.13 gives the data for the next four weeks. Figure 17.23 is the p chart.

TABLE 17.13 Proportions of Workers Absent During Four Weeks

Day	M	T	W	Th	F	M	T	W	Th	F
Workers absent	129	121	117	109	122	119	103	103	89	105
Proportion $\hat{p}$	0.131	0.123	0.119	0.110	0.124	0.121	0.104	0.104	0.090	0.106
Day	M	T	W	Th	F	M	T	W	Th	F
Workers absent	99	92	83	92	92	115	101	106	83	98
Proportion $\hat{p}$	0.100	0.093	0.084	0.093	0.093	0.117	0.102	0.107	0.084	0.099

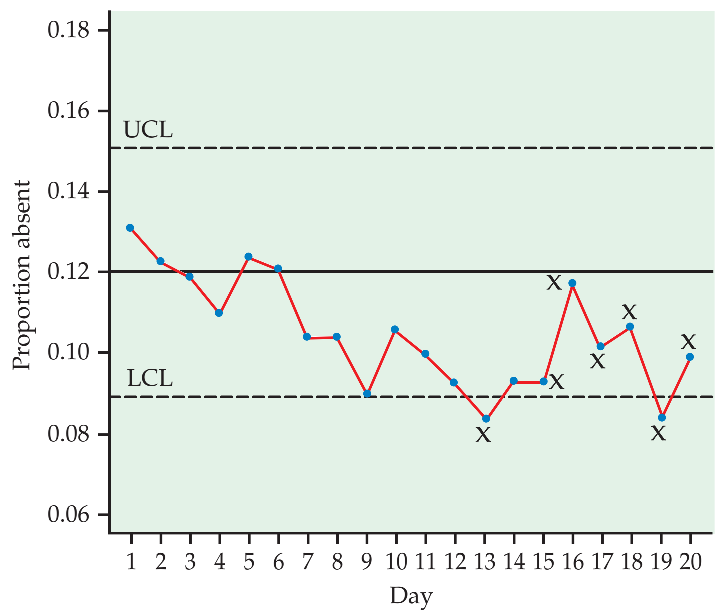


FIGURE 17.23 The *p* chart for daily proportion of workers absent over a four-week period, Example 17.21. The lack of control shows an improvement (decrease) in absenteeism. Update the chart to continue monitoring the process.

Figure 17.23 shows a clear downward trend in the daily proportion of workers who are absent. Days 13 and 19 lie below LCL, and a run of nine days below the center line is achieved at Day 15 and continues. Therefore, the points marked "x" are all out of control. It appears that a special cause (the various actions you took) has reduced the absenteeism rate from around 12% to around 10%. The last two weeks' data suggest that the rate has stabilized at this level. You will update the chart based on the new data. If the rate does not decline further (or even rises again as the effect of your actions wears off), you will consider further changes.

Example 17.21 is a bit oversimplified. The number of workers available did not remain fixed at 987 each day. Hirings, resignations, and planned vacations change the number a bit from day to day. The control limits for a day's $\hat{p}$ depend on n, the number of workers that day. If n varies, the control limits will move in and out from day to day. Software will do the extra arithmetic needed for a different n each day, but as long as the count of workers remains close to 987, the greater detail will not change your conclusion.

A single p chart for all workers is not the only, or even the best, choice in this setting. Because of the important role of supervisors in absenteeism, it would be wise to also keep separate p charts for the workers under each supervisor. These charts may show that you must reassign some supervisors.

SECTION 17.4 SUMMARY

- There are control charts for several different types of process measurements. One important type is the ***p* chart** for sample proportions $\hat{p}$.
- The interpretation of p charts is very similar to that of $\bar{x}$ charts. The out-of-control rules used are also the same.

SECTION 17.4 EXERCISES

17.72 Constructing a *p* chart for absenteeism. After inspecting Figure 17.23, you decide to monitor the next four weeks' absenteeism rates using a center line and control limits calculated from the second two weeks' data recorded in Table 17.13. Find $\bar{p}$ for these 10 days and give the new values of CL, LCL, and UCL. (Until you have more data, these are trial control limits. As long as you are taking steps to improve absenteeism, you have not reached the process-monitoring stage.)

17.73 Constructing a *p* chart for unpaid invoices. The controller's office of a corporation is concerned that invoices that remain unpaid after 30 days are damaging relations with vendors. To assess the magnitude of the problem, a manager searches payment records for invoices that arrived in the past 10 months. The average number of invoices is 2650 per month, with relatively little month-to-month variation. Of all these invoices, 958 remained unpaid after 30 days.

(a) What is the total number of opportunities for unpaid invoices? What is $\bar{p}$?

(b) Give the center line and control limits for a p chart on which to plot the future monthly proportions of unpaid invoices.

17.74 Constructing a *p* chart for mishandled baggage. The Department of Transportation reports that 3.09 of every 1000 passengers on domestic flights of the 10 largest U.S. airlines file a report of mishandled baggage.[17] Starting with this information, you plan to sample records for 2500 passengers per day at a large airport to monitor the effects of efforts to reduce mishandled baggage. What are the initial center line and control limits for a chart of the daily proportion of mishandled baggage reports? (You will find that LCL < 0. Because proportions $\hat{p}$ are always 0 or positive, take LCL $= 0$.)

17.75 Constructing a *p* chart for damaged eggs. An egg farm wants to monitor the effects of some new handling procedures on the percent of eggs arriving at the packaging center with cracked or broken shells. In the past, 2.18% of the eggs were damaged. A machine will allow the farm to inspect 500 eggs per hour. What are the initial center line and control limits for a chart of the hourly percent of damaged eggs?

17.76 More on constructing a *p* chart for damaged eggs. Refer to Exercise 17.75. Suppose that there are two machine operators, each working four-hour shifts. The first operator is very skilled and can inspect 500 eggs per hour. The second operator is less experienced and can inspect only 400 eggs per hour. Construct a p chart for an eight-hour day showing the appropriate center line and control limits.

17.77 Constructing a *p* chart for missing or deformed rivets. After completion of an aircraft wing assembly, inspectors count the number of missing or deformed rivets. There are hundreds of rivets in each wing, but the total number varies depending on the aircraft type. Recent data for wings with a total of 38,370 rivets show 194 missing or deformed. The next wing contains 1520 rivets. What are the appropriate center line and control limits for plotting the $\hat{p}$ from this wing on a p chart?

17.78 Constructing the *p* chart limits for incorrect or illegible prescriptions. A regional chain of retail pharmacies finds that about 1% of prescriptions it receives from doctors are incorrect or illegible. The chain puts in place a secure online system that doctors' offices can use to enter prescriptions directly. It hopes that fewer prescriptions entered online will be incorrect or illegible. A p chart will monitor progress. Use information about past prescriptions to set initial center line and control limits for the proportion of incorrect or illegible prescriptions on a day when the chain fills 90,000 online prescriptions. What are the center line and control limits for a day when only 45,000 online prescriptions are filled?

17.79 Calculating the *p* chart limits for school absenteeism. Here are data from an urban school district on the number of eighth-grade students with three or more unexcused absences from school during each month of a school year. Because the total number of eighth-graders changes a bit from month to month, these totals are also given for each month.

Month	Sept.	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May	June
Students	911	947	939	942	918	920	931	925	902	883
Absent	291	349	364	335	301	322	344	324	303	344

(a) Find $\bar{p}$. Because the number of students varies from month to month, also find $\bar{n}$, the average per month.

(b) Make a p chart using control limits based on $\bar{n}$ students each month. Comment on control.

(c) The exact control limits are different each month because the number of students n is different each month. This situation is common in using p charts. What are the exact limits for October and June, the months with the largest and smallest n? Add these limits to your p chart, using short lines spanning a single month. Do exact limits affect your conclusions?

17.80 *p* chart for a high-quality process. A manufacturer of consumer electronic equipment makes full use not only of statistical process control, but also of automated testing equipment that efficiently tests all completed products. Data from the testing equipment show that finished products have only 2.9 defects per million opportunities.

(a) What is $\bar{p}$ for the manufacturing process? If the process turns out 5000 pieces per day, how many defects do you expect to see per day? In a typical month of 24 working days, how many defects do you expect to see?

(b) What are the center line and control limits for a p chart for plotting daily defect proportions?

(c) Explain why a p chart is of no use at such high levels of quality.

17.81 More on monitoring a high-quality process. Because the manufacturing quality in the previous exercise is so high, the process of writing up orders is the major source of quality problems: the defect rate there is 8000 per million opportunities. The manufacturer processes about 500 orders per month.

(a) What is $\bar{p}$ for the order-writing process? How many defective orders do you expect to see in a month?

(b) What are the center line and control limits for a p chart for plotting monthly proportions of defective orders? What is the smallest number of bad orders in a month that will result in a point above the upper control limit?

CHAPTER 17 EXERCISES

17.82 Describing a process that is in control. A manager who knows no statistics asks you, "What does it mean to say that a process is in control? Is being in control a guarantee that the quality of the product is good?" Answer these questions in plain language that the manager can understand.

17.83 Constructing a Pareto chart. You manage the customer service operation for a maker of electronic equipment sold to business customers. Traditionally, the most common complaint is that equipment does not operate properly when installed, but attention to manufacturing and installation quality will reduce these complaints. You hire an outside firm to conduct a sample survey of your customers. Here are the percents of customers with each of several kinds of complaints:

Category	Percent
Accuracy of invoices	25
Clarity of operating manual	8
Complete invoice	24
Complete shipment	16
Correct equipment shipped	15
Ease of obtaining invoice adjustments/credits	33
Equipment operates when installed	6
Meeting promised delivery date	11
Sales rep returns calls	4
Technical competence of sales rep	12

(a) Why do the percents not add to 100%?

(b) Make a Pareto chart. What area would you choose as a target for improvement?

17.84 Choice of control chart. What type of control chart or charts would you use as part of efforts to assess quality? Explain your choices.

(a) Time to get security clearance.

(b) Percent of job offers accepted.

(c) Thickness of steel washers.

(d) Number of dropped calls per day.

17.85 Interpreting signals. Explain the difference in the interpretation of a point falling beyond the upper control limit of the $\bar{x}$ chart versus a point falling beyond the upper control limit of an s chart.

17.86 Selecting the appropriate control chart and limits. At the present time, about five out of every 1000 lots of material arriving at a plant site from outside vendors are rejected because they do not meet specifications. The plant receives about 350 lots per week. As part of an effort to reduce errors in the system of placing and filling orders, you will monitor the proportion of rejected lots each week. What type of control chart will you use? What are the initial center line and control limits?

You have just installed a new system that uses an interferometer to measure the thickness of polystyrene film. To control the thickness, you plan to measure three film specimens every 10 minutes and keep $\bar{x}$ and s charts. To establish control, you measure 22 samples of three films each at 10-minute intervals. Table 17.14 gives $\bar{x}$ and s for these samples. The units are millimeters $\times 10^{-4}$. Exercises 17.87 through 17.91 are based on this process improvement setting.

17.87 Constructing the s chart. Calculate control limits for s, make an s chart, and comment on control of short-term process variation. THICK

17.88 Recalculating the $\bar{x}$ and s charts. Interviews with the operators reveal that in Samples 1 and 10 mistakes in operating the interferometer resulted in one high-outlier thickness reading that was clearly incorrect. Recalculate $\bar{x}$ and s after removing Samples 1 and 10. Recalculate UCL for the s chart and add the new UCL to your s chart from the previous exercise. Control for the remaining samples is excellent. Now find the appropriate center line and control limits for an $\bar{x}$ chart, make the $\bar{x}$ chart, and comment on control. THICK

TABLE 17.14 $\bar{x}$ and s for Samples of Film Thickness (mm$\times 10^{-4}$)

Sample	$\bar{x}$	s	Sample	$\bar{x}$	s
1	848	20.1	12	823	12.6
2	832	1.1	13	835	4.4
3	826	11.0	14	843	3.6
4	833	7.5	15	841	5.9
5	837	12.5	16	840	3.6
6	834	1.8	17	833	4.9
7	834	1.3	18	840	8.0
8	838	7.4	19	826	6.1
9	835	2.1	20	839	10.2
10	852	18.9	21	836	14.8
11	836	3.8	22	829	6.7

17.89 Capability of the film thickness process. The specifications call for film thickness 830 ± 25 $mm \times 10^{-4}$. THICK

(a) What is the estimate $\hat{\sigma}$ of the process standard deviation based on the sample standard deviations (after removing Samples 1 and 10)? Estimate the capability ratio C_p and comment on what it says about this process.

(b) Because the process mean can easily be adjusted, C_p is more informative than C_{pk}. Explain why this is true.

(c) The estimate of C_p from part (a) is probably too optimistic as a description of the film produced. Explain why.

17.90 Calculating the percent that meet specifications. Examination of individual measurements shows that they are close to Normal. If the process mean is set to the target value, about what percent of films will meet the specifications? THICK

17.91 More on the film thickness process. Previously, control of the process was based on categorizing the thickness of each film inspected as satisfactory or not. Steady improvement in process quality has occurred, so that just 15 of the last 5000 films inspected were unsatisfactory. THICK

(a) What type of control chart would be used in this setting, and what would be the control limits for a sample of 100 films?

(b) The chart in part (a) is of little practical value at current quality levels. Explain why.

17.92 Probability of an out-of-control signal. There are other out-of-control rules that are sometimes used with $\bar{x}$ charts. One is "15 points in a row within the 1σ level." That is, 15 consecutive points fall between $\mu - \sigma/\sqrt{n}$ and $\mu + \sigma/\sqrt{n}$. This signal suggests either that the value of σ used for the chart is too large or that careless measurement is producing results that are suspiciously close to the target. Find the probability that the next 15 points will give this signal when the process remains in control with the given μ and σ.

17.93 Probability of another out-of-control signal. Another out-of-control signal is when four out of five successive points are on the same side of the center line and farther than $\sigma/\sqrt{n}$ from it. Find the probability of this event when the process is in control.

CHAPTER 17 NOTES AND DATA SOURCES

1. Texts on quality management give more detail about these and other simple graphical methods for quality problems. The classic reference is Kaoru Ishikawa, *Guide to Quality Control,* Asian Productivity Organization, 1986.

2. The flowchart and a more elaborate version of the cause-and-effect diagram for Example 17.3 were prepared by S. K. Bhat of the General Motors Technical Center as part of a course assignment at Purdue University.

3. Walter Shewhart's classic book, *Economic Control of Quality of Manufactured Product* (Van Nostrand, 1931), organized the application of statistics to improving quality.

4. We have adopted the terms "chart setup" and "process monitoring" from Andrew C. Palm's discussion of William H. Woodall, "Controversies and contradictions in statistical process control," *Journal of Quality Technology,* 32 (2000), pp. 341–350. Palm's discussion appears in the same issue, pp. 356–360. We have combined Palm's stages B ("process improvement") and C ("process monitoring") in writing for beginners because the distinction between them is one of degree.

5. It is common to call these "standards given" $\bar{x}$ and s charts. We avoid this term because it easily leads to the common and serious error of confusing control limits (based on the process itself) with standards or specifications imposed from outside.

6. Data provided by Charles Hicks, Purdue University.

7. See, for example, Chapter 3 of Stephen B. Vardeman and J. Marcus Jobe, *Statistical Quality Assurance Methods for Engineers,* Wiley, 1999.

8. The classic discussion of out-of-control signals and the types of special causes that may lie behind special control chart patterns is the *AT&T Statistical Quality Control Handbook,* Western Electric, 1956.

9. The data in Table 17.6 are adapted from data on viscosity of rubber samples appearing in Table P3.3 of Irving W. Burr, *Statistical Quality Control Methods,* Marcel Dekker, 1976.

10. The control limits for the s chart based on past data are commonly given as $B_4\bar{s}$ and $B_3\bar{s}$. That is, $B_4 = B_6/c_4$ and $B_3 = B_5/c_4$. This is convenient for users, but we choose to minimize the number of control chart constants students must keep straight and to emphasize that process-monitoring and past-data charts are exactly the same except for the source of μ and σ.

11. Simulated data based on information appearing in Arvind Salvekar, "Application of six sigma to DRG 209," found at the Smarter Solutions website, **www.smartersolutions.com**.

12. Data provided by Linda McCabe, Purdue University.

13. The first two Deming quotations are from *Public Sector Quality Report,* December 1993, p. 5. The third quotation is part of the 10th of Deming's "14 points of quality management," from his book *Out of the Crisis,* MIT Press, 1986.

14. Control charts for *individual measurements* cannot use within-sample standard deviations to estimate short-term process variability. The spread between successive observations is the next best thing. Texts such as that cited in Note 7 give the details.

15. The data in Figure 17.17(b) are simulated from a probability model for call pickup times. That pickup times for large financial institutions have median 20 seconds and mean 32 seconds is reported by Jon Anton, "A case study in benchmarking call centers," Purdue University Center for Customer-Driven Quality, no date.

16. These 2011 statistics can be found at **https://nces.ed.gov/pubs2011/2011033.pdf.**

17. Data obtained from "Air travel consumer report," *Office of Aviation Enforcement and Proceedings*, March 2013.