

Random Variables and Probability Models (Chapter 14)

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Introduction and background

This document is intended to help describe how to undertake analyses introduced as examples in the Fourth Edition of *Intro Stats* (2013) by De Veaux, Velleman, and Bock. More information about the book can be found at http://wps.aw.com/aw_deveaux_stats_series. This file as well as the associated R Markdown reproducible analysis source file used to create it can be found at <https://nhorton.people.amherst.edu/is4>.

This work leverages initiatives undertaken by Project MOSAIC (<http://www.mosaic-web.org>), an NSF-funded effort to improve the teaching of statistics, calculus, science and computing in the undergraduate curriculum. In particular, we utilize the `mosaic` package, which was written to simplify the use of R for introductory statistics courses. A short summary of the R needed to teach introductory statistics can be found in the `mosaic` package vignettes (<http://cran.r-project.org/web/packages/mosaic>). A paper describing the `mosaic` approach was published in the *R Journal*: <https://journal.r-project.org/archive/2017/RJ-2017-024>.

Note that some of the figures in this document may differ slightly from those in the IS4 book due to small differences in datasets. However in all cases the analysis and techniques in R are accurate.

Chapter 14: Random Variables and Probability Models

Section 14.1: Center (the Expected Value)

We can replicate the calculation on page 363:

```
library(mosaic); library(readr); options(digits=3)
x <- c(10000, 5000, 0)
prob <- c(1/1000, 2/1000, 997/1000)
sum(prob) # sums to 1
```

```
## [1] 1
```

```
expect <- sum(x*prob); expect # expected value
```

```
## [1] 20
```

Section 14.2: Spread (The Standard Deviation)

We can continue with the example from page 365:

```
xminmu <- x - expect; xminmu
```

```
## [1] 9980 4980 -20
```

```
myvar <- sum(xminmu^2*prob); myvar
```

```
## [1] 149600
```

```
sd <- sqrt(myvar); sd
```

```
## [1] 387
```

Section 14.3: Combining Random Variables

Let's replicate the values from the example on page 367:

```
ex <- 5.83; varx <- 8.62^2  
ed <- ex+5; ed
```

```
## [1] 10.8
```

```
vard <- varx; vard
```

```
## [1] 74.3
```

```
sqrt(vard)
```

```
## [1] 8.62
```

Section 14.4: The Binomial Model

Section 14.5: Modeling the Binomial with a Normal Model

Section 14.6: The Poisson Model

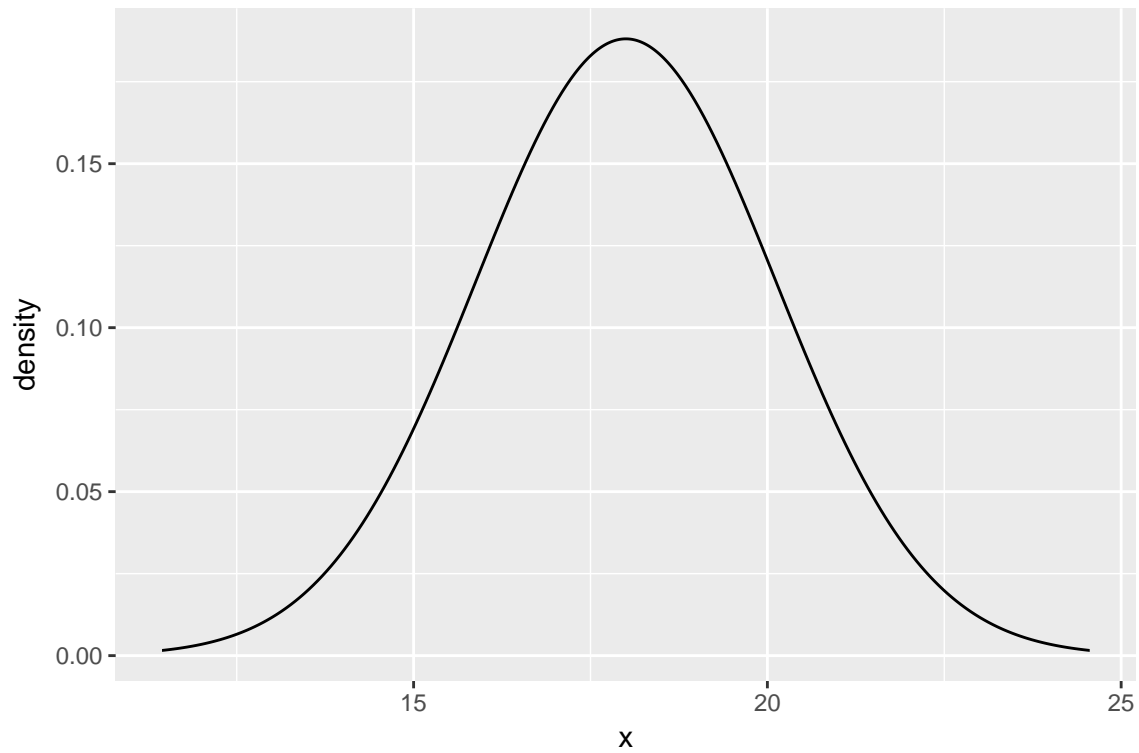
Section 14.7: Continuous Random Variables

Let's replicate the Think/Show/Tell/Conclusion on pages 381 and 382:

```
sdval <- sqrt(4.50); sdval
```

```
## [1] 2.12
```

```
gf_dist("norm", params=list(18, sdval), xlab="x", ylab="f(x)")
```



```
xpnorm(20, mean=18, sd=sdval) # note how exact value is different from the table!
```

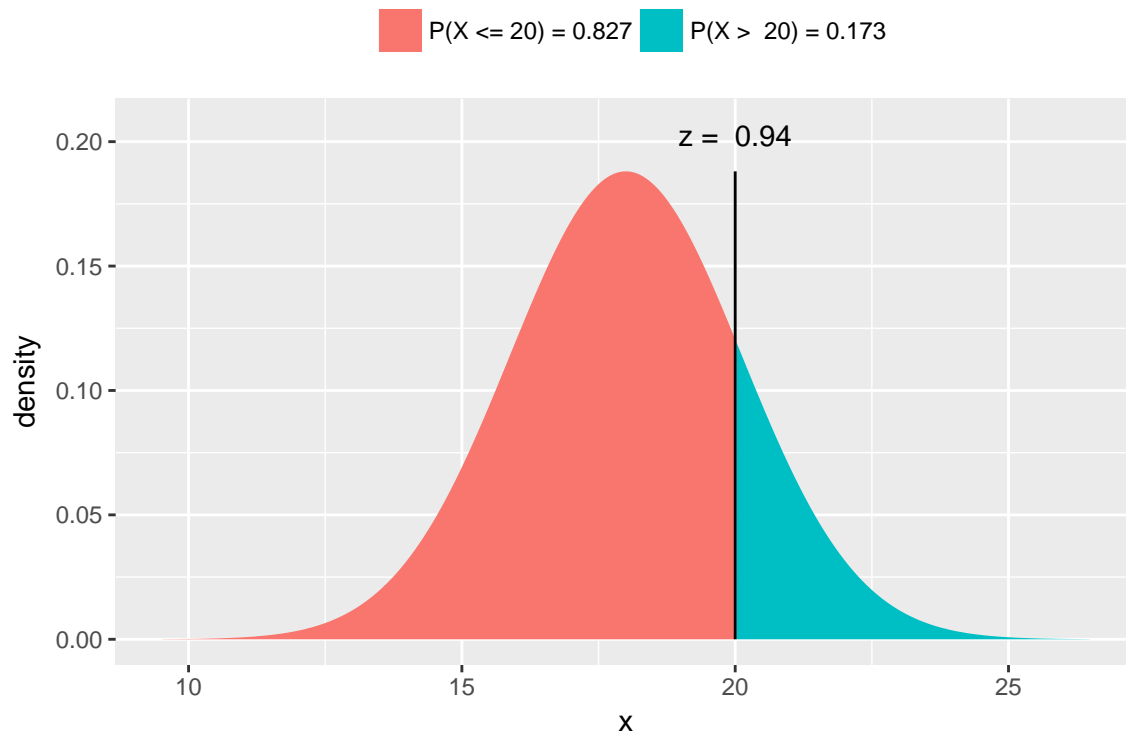
```
##
```

```
## If  $X \sim N(18, 2.121)$ , then
```

```
##  $P(X \leq 20) = P(Z \leq 0.9428) = 0.8271$ 
```

```
##  $P(X > 20) = P(Z > 0.9428) = 0.1729$ 
```

```
##
```



```
## [1] 0.827
```

```
zval <- (20-18)/sdval; zval
```

```
## [1] 0.943
```

```
xpnorm(zval, mean=0, sd=1)
```

```
##
```

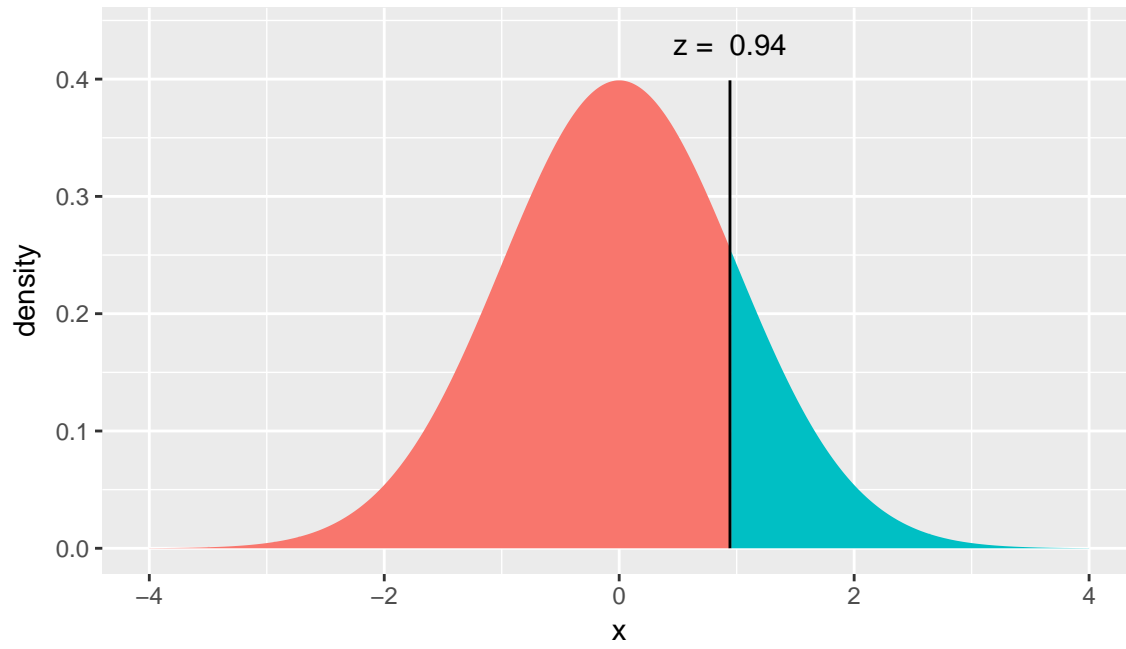
```
## If  $X \sim N(0, 1)$ , then
```

```
##  $P(X \leq 0.9428) = P(Z \leq 0.9428) = 0.8271$ 
```

```
##  $P(X > 0.9428) = P(Z > 0.9428) = 0.1729$ 
```

```
##
```

■ $P(X \leq 0.94) = 0.827$ ■ $P(X > 0.94) = 0.173$



```
## [1] 0.827
```